**Mini Lecture 1.1**

Introduction to Algebra: Variables and Mathematical Models

**Learning Objectives:**

1. Evaluate algebraic expressions.
2. Translate English phrases into algebraic expressions.
3. Determine whether a number is a solution of an equation.
4. Translate English sentences into algebraic equations.
5. Evaluate formulas.

**Examples:**

1. Evaluate each expression for $x = 5$.

   a. $4(x-3)$ b. $\dfrac{6x-15}{3x}$

2. Evaluate each expression for $x = 3$ and $y = 6$.

   a. $5(x+y)$ b. $\dfrac{2x+3y}{2y}$

3. Write each English phrase as an algebraic expression. Let $x$ represent the number.
   a. the difference of a number and six
   b. eight more than four times a number
   c. four less than the quotient of a number and twelve

4. Determine whether the given number is a solution of the equation.
   a. $x - 8 = 12$; 20 b. $4x - 7 = 9$; 3 c. $3(y - 5) = 6$; 7

5. Write each English sentence as an equation. Let $x$ represent the number.
   a. The product of a number and seven is twenty-one.
   b. The difference of twice a number and three is equal to twenty-seven.
   c. Six less than three times a number is the same as the number increased by twelve.

**Teaching Notes:**

- It may be helpful to draw students' attention to the word "evaluate." Help them see the letters v – a – l – u . This will help them remember that evaluate means to find the value of an expression.
- Students often make mistakes with the phrase "less than" so they should be cautioned about the order of the subtraction.
- Translating from English to algebra is an important skill that will be used often.

Answers: 1a. 8 b. 1 2a. 45 b. 2 3a. $x - 6$ b. $4x + 8$ c. $\dfrac{x}{12} - 4$ 4a. yes b. not a solution c. yes 5a. $7x = 21$ b. $2x - 3 = 27$ c. $3x - 6 = x + 12$

**Mini Lecture 1.2**
Fractions in Algebra

**Learning Objectives:**

1. Convert between mixed numbers and improper fractions.
2. Write the prime factorization of a composite number.
3. Reduce or simplify fractions.
4. Multiply fractions.
5. Divide fractions.
6. Add and subtract fractions with identical denominators.
7. Add and subtract fractions with unlike denominators.
8. Solve problems involving fractions in algebra.

**Examples:**

1. Convert each mixed number to an improper fraction.

a. $3\frac{7}{10}$ b. $8\frac{3}{7}$ c. $5\frac{2}{3}$ d. $9\frac{1}{4}$

2. Convert each improper fraction to a mixed number.

a. $\frac{13}{8}$ b. $\frac{12}{11}$ c. $\frac{25}{3}$ d. $\frac{37}{7}$

3. Give the prime factorization of each of the following composite numbers.

a. 24 b. 48 c. 90 d. 108

4. What makes a number a prime?

5. Reduce the following fractions to lowest terms by factoring each numerator and denominator and dividing out common factors.

a. $\frac{10}{12}$ b. $\frac{32}{48}$ c. $\frac{24}{50}$ d. $\frac{77}{98}$

6. Perform the indicated operation. Always reduce answer, if possible.

a. $\frac{3}{4}+\frac{1}{6}$ b. $8\frac{1}{8}+3\frac{1}{3}$ c. $\frac{7}{10}-\frac{3}{8}$

d. $10\frac{11}{12}-4\frac{1}{4}$ e. $\left(\frac{7}{9}\right)\left(\frac{18}{19}\right)$ f. $\left(6\frac{2}{3}\right)\left(2\frac{1}{4}\right)$

g. $\frac{7}{8}\div\frac{3}{4}$ h. $5\frac{3}{8}\div 2\frac{1}{4}$

**Teaching Notes:**

- When teaching factorization, it is often helpful to review divisibility rules.
- To add or subtract fractions, you must have a LCD.
- To divide fractions, multiply by the reciprocal of the divisor.
- To multiply or divide mixed numbers, change to improper fractions first.

<u>Answers:</u> 1. a. 37/10 b. 59/7 c. 17/3 d. 37/4 2. a. 1 5/8 b. 1 1/11 c. 8 1/3 d. 5 2/7
3. a. $2 \cdot 2 \cdot 2 \cdot 3$ b. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ c. $2 \cdot 3 \cdot 3 \cdot 5$ d. $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
4. a number whose only factors are 1 and itself 5. a. 5/6 b. 2/3 c. 12/25 d. 11/14 6. a. 11/12
b. $\frac{275}{24}$ or 11 11/24 c. 13/40 d. $\frac{20}{3}$ or 6 2/3 e. 14/19 f. 15 g. 7/6 or 1 1/6 h. 43/18 or 2 7/18

## Mini Lecture 1.3
The Real Numbers

**Learning Objectives:**

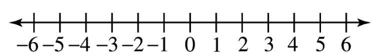
1. Define the sets that make up the set of real numbers.
2. Graph numbers on a number line.
3. Express rational numbers as decimals.
4. Classify numbers as belonging to one or more sets of the real numbers.
5. Understand and use inequality symbols.
6. Find the absolute value of a real number.

**Examples:**

1. Answer the following questions about each number:
   Is it a natural number? Is it rational?
   Is it a whole number? Is it irrational?
   Is it an integer? Is it a real number?

a. 18  b. –3.5  c. $\sqrt{5}$  d. 0  e. $-\frac{3}{4}$  f. $\pi$  g. –5  h. 0.45

2. Graph each number on the number line.




a. 5.5  b. $-\frac{16}{4}$  c. $2\frac{1}{4}$  d. –3.2

3. Express each rational number as a decimal.

a. $\frac{7}{8}$  b. $\frac{9}{11}$  c. $\frac{5}{3}$  d. $\frac{1}{4}$

4. Use $>$ or $<$ to compare the numbers.

a. 18 □ –20  b. –16 □ –13  c. –4.3 □ –6.2

d. $\frac{4}{7}$ □ $\frac{8}{11}$  e. $-\frac{3}{5}$ □ $\frac{2}{3}$

5. Give the absolute value.

a. $|8|$  b. $|-5|$  c. $|-3.2|$  d. $|22|$

**Teaching Notes:**

- Make sure the students have minimal understanding of square roots.
- Absolute value is ALWAYS POSITIVE because it measures distance from zero.
- Remind students that a number cannot be rational and irrational.
- To change a rational number to a decimal, divide the numerator by the denominator.

<u>Answers</u>: 1. a. natural, whole, integer, rational , real  b. rational, real  c. irrational, real  d. whole, integer, rational, real  e. rational, real  f. irrational, real  g., integer, rational, real  h. rational, real  2. See below  3. a. 0.875  b. 0.81  c. 0.6  d. 0.25  4. a. $>$  b. $<$  c. $>$  d. $<$  e. $<$  5. a. 8  b. 5  c. 3.2  d. 22

**Mini Lecture 1.4**
Basic Rules of Algebra

**Learning Objectives:**

1. Understand and use the vocabulary of algebraic expressions.
2. Use commutative properties.
3. Use associative properties.
4. Use the distributive property.
5. Combine like terms.
6. Simplify algebraic expressions.

**Examples:**

1. Fill in the blanks.

| Algebraic Expression | # of terms | coefficients | like terms |
|---|---|---|---|
| a. $6y - 3x - 4y + 8$ | ________ | ___________ | ________ |
| b. $5x^2 + 2y - 2x^2 + 9 - 3y$ | ________ | ___________ | ________ |
| c. $6x^2 - 9y + 4x + 8 - y + 5$ | ________ | ___________ | ________ |

2. Name the property being illustrated and then simplify if possible.
   a. $6(x + 2) = 6x + 12$ ____________________________________

   b. $(9 \bullet 12)5 = 9(12 \bullet 5)$ ____________________________________

   c. $(x + 4) + 8 = x + (4 + 8)$ ____________________________________

   d. $(2)(3.14)(5) = 2(5)(3.14)$ ____________________________________

3. Simplify.
   a. $6x - x + 2x =$ _____________ b. $3a - 8 + 2a + 10 =$ ________________
   c. $6(x + 3) - 5 =$ ____________ d. $2(x - 4) - (x - 2) =$ _______________
   e. $5(y - 2) + 3(4 - y) =$ _____________

**Teaching Notes:**

- A coefficient is the number factor of a term.
- Like terms have the very same variables raised to the same exponents.
- When applying the commutative property, only the order changes.
- The commutative property holds for addition and multiplication only.
- When applying the associative property the grouping changes.
- The associative property holds for addition and multiplication only.
- When combining like terms, add or subtract the coefficients, the variable part remains the same.
- Always use parentheses when substituting a value for a variable.

Answers: 1 a. 4; 6, –3, –4, 8; $6y$ and $-4y$ b. 5; 5, 2, –2, 9, –3; $5x^2$ and $-2x^2$; $2y$ and $-3y$
c. 6; 6, –9, 4, 8, –1, 5; $9y$ and $-y$; 8 and 5 2. a. distributive b. associative of multiplication
c. associative of addition d. commutative of multiplication 3. a. $7x$ b. $5a + 2$ c. $6x + 13$
d. $x - 6$ e. $2y + 2$

**Mini Lecture 1.5**
Addition of Real Numbers

**Learning Objectives:**

1. Add numbers with a number line.
2. Find sums using identity and inverse properties.
3. Add numbers without a number line.
4. Use addition rules to simplify algebraic expressions.
5. Solve applied problems using a series of additions.

**Examples:**

1. Find each sum using a number line.

   a. $3 + -5$ b. $-4 + -6$ c. $-1 + 2$ d. $5 + 4$

2. Add without using a number line.

   a. $-7 + -11$ b. $-0.4 + -3.2$ c. $-\frac{4}{5} + -\frac{3}{10}$ d. $-15 + 4$

   e. $7.1 + 8.5$ f. $-8 + 25$ g. $-6.4 + 6.1$ h. $\frac{5}{8} + -\frac{3}{4}$

3. Simplify the following.

   a. $-30x + 5x$ b. $-2y + 5x + 8x + 3y$ c. $-2(3x + 5y) + 6(x + 2y)$

4. Write a sum of signed numbers that represents the following situation. Then, add to find the overall change.

   If the stock you purchased last week rose 2 points, then fell 4, rose 1, fell 2, and rose 1, what was the overall change for the week?

**Teaching Notes:**

- When adding numbers with like signs, add and take the sign.
- When adding numbers with unlike signs, subtract the smaller absolute value from the larger absolute value, and the answer will have the sign of the number with the larger absolute value.

Answers: 1. a. $-2$ b. $-10$ c. $1$ d. $9$ 2. a. $-18$ b. $-3.6$ c. $-\frac{11}{10}$ or $-1\frac{1}{10}$ d. $-11$ e. $15.6$ f. $17$ g. $-0.3$ h. $-\frac{1}{8}$ 3. a. $-25x$ b. $13x + y$ c. $2y$
4. $2 + (-4) + 1 + (-2) + 1 = -2$; fell 2 points

**Mini Lecture 1.6**
Subtraction of Real Numbers

**Learning Objectives:**

1. Subtract real numbers.
2. Simplify a series of additions and subtractions.
3. Use the definition of subtraction to identify terms.
4. Use the subtraction definition to simplify algebraic expressions.
5. Solve problems involving subtraction.

**Examples:**

1. Subtract by changing each subtraction to addition of the opposite first.

   a. 6 – 12 b. –15 – 15 c. 13 – 21 d. $\frac{2}{5}-\frac{5}{6}$
   e. 4.2 – 6.8 f. 25 – (–25) g. –51 – (–13) h. 14 – (–13)

2. Simplify.
   a. –16 – 14 – (–10) b. –20.3 – (–40.1) – 18
   c. 15 – (–3) – 10 – 18 d. –11 – 21 – 31 – 41

3. Identify the number of terms in each expression; then name the terms.
   a. $4x - 6y + 12 - 3y$ b. $16 - 2x - 15$
   c. $15a - 2ab + 3b - 6a + 18$ d. $5y - x + 3y - 14xy$

4. Simplify each algebraic expression.
   a. $8x + 7 - x$ b. $-11y - 14 + 2y - 10$
   c. $15a - 10 - 12a + 12$ d. $25 - (-3x) - 15 - (-2x)$

5. Applications.
   a. The temperature at dawn was –7 degrees but fortunately the sun came out and by 4:00 p.m. the temperature had reached 38 degrees. What was the difference in the temperature at dawn and 4:00 p.m.?
   b. Express 214 feet below sea level as a negative integer. Express 10,510 above sea level as a positive integer. What is the difference between the two elevations?

**Teaching Notes:**

- Say the problem to yourself. When you hear the word “minus”, immediately make a “change-change”. That means to “change” the subtraction to addition and “change” the sign of the number that follows to its opposite.
- Remember, the sign in front of a term goes with the term.
- The symbol “–”can have different meanings:
  1. subtract or “minus” only when it is between 2 terms
  2. the opposite of
  3. negative

Answers: 1. a. –6 b. –30 c. –8 d. $-\frac{13}{30}$ e. –2.6 f. 50 g. –38 h. 27 2. a. –20 b. 1.8 c. –10 d. –104 3. a. 4 terms; $4x, -6y$, 12, $-3y$ b. 3 terms; 16, $-2x$ , –15 c. 5 terms; $15a$, $-2ab$, $3b$, $-6a$, 18 d. 4 terms; $5y$, $-x$, $3y$, $-14xy$ 4. a. $7x + 7$ b. $-9y - 24$ c. $3a + 2$ d. $5x + 10$ 5. a. 45 degrees b. –214 feet. 10,500 feet; 10, 724 feet

## Mini Lecture 1.7

Multiplication and Division of Real Numbers

**Learning Objectives:**

1. Multiply real numbers.
2. Multiply more than two real numbers.
3. Find multiplicative inverses.
4. Use the definition of division.
5. Divide real numbers.
6. Simplify algebraic expressions involving multiplication.
7. Determine whether a number is a solution of an equation.
8. Use mathematical models involving multiplication and division.

**Examples:**

1. Multiply.

   a. $(3)(-4)$  b. $(-6)(-5)$  c. $(-8)(0)$  d. $(-3.2)(-1.1)$  e. $\left(-\frac{3}{4}\right)\left(\frac{2}{9}\right)$

   f. $(-5)(2)(-1)$  g. $(-2)(2)(-3)(-3)$

2. Find the multiplicative inverse of each number.

   a. $-8$  b. $\frac{2}{5}$  c. $-7$  d. $\frac{1}{4}$

3. Use the definition of division to find each quotient.

   a. $-49 \div 7$  b. $\frac{-24}{-4}$

4. Divide or state that the expression is undefined.

   a. $\frac{-18}{0}$  b. $-\frac{4}{5} \div \frac{20}{25}$  c. $-32.4 \div 8$  d. $0 \div -8$

5. Simplify.

   a. $-3(2x)$  b. $9x + x$  c. $-12a + 4a$  d. $-(5x - 3)$

   e. $-2(3y + 4)$  f. $2(3x + 4) - (4x - 6)$

**Teaching Notes:**

- The product of an even number of negative numbers is positive.
- The product of an odd number of negative numbers is negative.
- Any product using zero as a factor will equal zero.
- The quotient of two real numbers with different signs is negative.
- The quotient of two real numbers with same signs is positive.
- Division of a non-zero number by zero is undefined.
- Any non-zero number divided into 0 is 0.

Answers: 1. a. $-12$  b. 30  c. 0  d. 3.52  e. $-\frac{1}{6}$  f. 10  g. $-36$  2. a. $-\frac{1}{8}$  b. $\frac{5}{2}$  c. $-\frac{1}{7}$  d. $\frac{4}{1}$
3. a. $-7$  b. 6  4. a. undefined  b. $-1$  c. $-4.05$  d. 0  5.a. $-6x$  b. $10x$  c. $-8a$  d. $-5x + 3$
e. $-6y - 8$  f. $2x + 14$

**Mini Lecture 1.8**
Exponents and Order of Operations

**Learning Objectives:**

1. Evaluate exponential expressions.
2. Simplify algebraic expressions with exponents.
3. Use order of operations agreement.
4. Evaluate mathematical models.

**Examples:**

1. Identify the base and the exponent, then evaluate.
   a. $3^4$ b. $(-4)^3$ c. $-8^2$ d. $(-8)^2$

2. Evaluate.
   a. $13^2$ b. $2^5$ c. $(-3)^3$ d. $5^2$

3. Simplify if possible.
   a. $6x^2 - x^2$ b. $5y^3 + 2y - 3y^3$ c. $6a^2 + 2a - 4a^2 - 6a$
   d. $10p^3 - 8p^2$

4. Simplify by using the order of operations.
   a. $30 \div 2 \cdot 3 - 52$ b. $14 - (33 \div 11) + 4$
   c. $(5 + 2)^2$ d. $10 - 7(32 \div 8) + 5 \cdot 3$
   e. $\left(\frac{1}{4}\right)+\left(\frac{1}{3}\right)^2$ f. $15 - 3[8 - (-12 \div 2^2) - 4^2]$
   g. $\dfrac{16+4^2 \div 8}{-2-(-5)}$ h. $22 + 5(x + 7) - 3x - 10$

5. 5. Evaluate each expression for the given value.
   a. $-a - a^2$ if $a = -3$ b. $-a - a^2$ if $a = 3$ c. $4x^2 - x + 3x$ if $x = -1$

6. Use the formula for perimeter of a rectangle, $P = 2w + 2l$ to find the perimeter of a rectangle if the length is 28 cm and the width is 15 cm.

**Teaching Notes:**

- If the negative sign is part of the base, it will be inside the parentheses.
- **NEVER** multiply the base and the exponent together.
- The exponent tells how many times to write the base as a factor.
- Always use parentheses when substituting a value for a variable.
- The Order of Operations must be followed on every problem.

Answers: 1. a. 81 b. –64 c. –64 d. 64 2. a. 169 b. 32 c. –27 d. 25 3. a. $5x^2$ b. $2y^3 + 2y$ c. $2a^2 - 4a$ d. $10p^3 - 8p^2$ 4. a. –7 b. 15 c. 49 d. –3 e. $\frac{13}{36}$ f. 30 g. 6 h. $2x + 47$ 5. a. –6 b. –12 c. 2 6. 86 cm

**Mini Lecture 2.1**
The Addition Property of Equality

**Learning Objectives:**

1. Identify linear equations in one variable.
2. Use the addition property of equality to solve equations.
3. Solve applied problems using formulas.

**Examples:**

1. Identify the linear equations in one variable.
   a. $x+7=10$
   b. $x^2-2=7$
   c. $\frac{3}{x}=5$
   d. $|x+1|=6$
2. Solve the following equations using the addition property of equality. Be sure to check your proposed solution.
   a. $x+2=17$

   b. $-12=x-9$

   c. $x-\frac{1}{2}=4$

   d. $3x-2x=8$

   e. $5x+1=4(x-2)$

   f. $x+3.5=4.8$

   g. $2x+5=x-2$

   h. $3x+5=2x+5$

3. If Sue is 2 years older than John then we will use S to represent Sue's age and J to represent John's age. Use the equation $S=J+2$ to find John's age if Sue is 41.

**Teaching Notes:**

- Solving an equation is the process of finding the number (or numbers) that make the equation a true statement. These numbers are called the solutions, or roots, or the equation.
- To apply the addition property of equality, one must add the same number or expression to both sides of the equation.
- Equivalent equations are equations that have the same solution.

*Answers:* 1. a. linear b. not linear c. not linear d. not linear 2. a. 15 b. -3 c. $4\frac{1}{2}$ or $\frac{9}{2}$ d. 8 e. -9 f. 1.3 g. -7 h. 0 3. 39

## Mini Lecture 2.2
The Multiplication Property of Equality

**Learning Objectives:**

1. Use multiplication property of equality to solve equations.
2. Solve equations in the form $-x = c$.
3. Use addition and multiplication properties to solve equations.
4. Solve applied problems using formulas.

**Examples:**

1. Multiply both sides of the equation by the reciprocal of the coefficient of the variable to solve for the variable.

   a. $\frac{x}{3} = 6$    b. $\frac{x}{-2} = -7$    c. $\frac{y}{15} = -10$    d. $8 = \frac{x}{-3}$

2. Divide both sides of the equation by the coefficient of the variable to solve for the variable.

   a. $6x = 18$    b. $-2x = -14$    c. $15y = -10$    d. $24 = -3x$

Both of the above methods of isolating the variable are effective for solving equations.

3. Solve each equation by multiplying or dividing.

   a. $18y = -108$    b. $\frac{3}{5}x = 12$    c. $124 = \frac{x}{3}$    d. $-7x = -63$

4. Multiply or divide both sides of each equation by –1 to get a positive $x$.

   a. $-x = -7$    b. $82 = -x$    c. $-a = -\frac{3}{7}$    d. $14 = -x$

5. Solve each equation using both the addition and multiplication properties of equality.

   a. $3x - 5 = 13$    b. $18 - 6x = 14 - 2x$    c. $23 = 2a - 7$

   d. $-6y - 21 = 21$    e. $33 - x = 3x - 11$    f. $\frac{2}{3}x - 6 = 12$

**Teaching Notes:**

- Remind students that reciprocals always have the same sign.
- When students see $-x$ they must realize the coefficient is –1.

Answers: 1. a. $x = 18$ b. $x = 14$ c. $y = -150$ d. $x = -24$ 2. a. $x = 3$ b. $x = 7$ c. $y = -\frac{2}{3}$ d. $x = -8$

3. a. $y = -6$ b. $x = 20$ c. $x = 372$ d. $x = 9$ 4. a. $x = 7$ b. $x = -82$ c. $a = \frac{3}{7}$ d. $x = -14$

5. a. $x = 6$ b. $x = 1$ c. $a = 15$ d. $y = -7$ e. $x = 11$ f. $x = 27$

**Mini Lecture 2.3**
Solving Linear Equations

**Learning Objectives:**

1. Solve linear equations.
2. Solve linear equations containing fractions.
3. Solve linear equations containing decimals.
4. Identify equations with no solution or infinitely many solutions.
5. Solve applied problems using formulas.

**Examples:**

1. $3x + 2x + 8 = -7 + x + 11$
2. $6x = 3(x + 9)$
3. $5(2x - 1) - 15 = 3(4x + 2) + 4$
4. $\frac{x}{5} = \frac{2x}{3} + \frac{7}{15}$
5. $1.2x + 1.8 = 0.6x$
6. $1.3x + 1.7 = -1 - 1.4x$
7. $2x + 9 = 2(x + 4)$
8. $4(x + 2) + 5 = 5(x + 1) + 8$
9. Use the formula $P = 4s$ to find the length of a side of a square whose perimeter is 32 in.

**Teaching Notes:**

- Simplify the algebraic expression on each side of the equal sign.
- Collect variable terms on one side of the equal sign and all constant terms on the other side of the equal sign.
- Isolate the variable and solve.
- Check your solution in the original expression.

Answers: 1. –1 2. 9 3. –15 4. –1 5. –3 6. –1 7. inconsistent, no solution 8. 0 9. 8 inches

## Mini Lecture 2.4

Formulas and Percents

**Learning Objectives:**

1. Solve a formula for a variable.
2. Use the percent formula.
3. Solve applied problems involving percent change.

**Examples:**

1. Solve the formula for the indicated variable by isolating the variable.

   a. $A = \dfrac{B_1 + B_2}{2}$ for $B_1$ b. $P = a + \mathrm{b} + c$ for $c$

   c. $A = \pi r^2 h$ for $h$ d. $4p + H = M$ for $p$

   e $Ax + By = C$ for $A$ f. $y = mx + b$ for $b$

2. Translate each question into an equation using the percent formula, $A = PB$, then solve the equation.

   a. What is 15 percent of 60? b. 62% of what number is 31?

   c. What percent of 132 is 33? d. 60 is what percent of 500?

3. The average, or mean $A$ of the three exam grades, $x$, $y$, $z$, is given by formula $A = \dfrac{x + y + z}{3}$.

   a. Solve the formula for $z$.

   b. If your first two exams are 75% and 83% ($x = 75$, $y = 83$), what must you get on the third exam to have an average of 80%?

**Teaching Notes:**

- Many students have trouble solving formulas for a letter and need to be reminded the same steps are used when solving for a letter in a formula as are used when solving any equation for a variable.
- When changing a decimal to a percent, move the decimal point two places to the right and use the % symbol.
- When changing a percent to a decimal, move the decimal point two places to the left and drop the % symbol.
- When translating English into a mathematical equation, the word "is" translates to equals and the word "of" means multiply.

Answers: 1. a. $B_1 = 2A - B_2$ b. $c = P - a - b$ c. $h = \dfrac{A}{\pi r^2}$ d. $p = \dfrac{M - H}{4}$ e. $A = \dfrac{C - By}{x}$
f. $b = y - mx$ 2. a. $x = 0.15(60)$; 9 b. $0.62x = 31$; 50 c. $x \cdot 132 = 33$; 25% d. $60 = x \cdot 500$; 12%
3. a. $z = 3A - x - y$ b. 82%

**Mini Lecture 2.5**
An Introduction to Problem Solving

**Learning Objectives:**
1. Translate English phrases into algebraic expressions.
2. Solve algebraic word problems using linear equations.

**Examples:**
1. Translate each English phrase into an algebraic expression. Let "$x$" represent the unknown.
   a. Three times a number decreased by 11.
   b. The product of seven and a number increased by 2.
   c. Eight more than a number.

2. Translate each sentence into an algebraic equation and then solve the equation.
   a. Twice a number less five is eleven.
   b. Five times the sum of a number and eight is 30.

3. Identify all unknowns, set up an equation, and then solve.
   a. Bill earns five dollars more per hour than Joe. Together their pay for one hour totals $21. How much does each man earn per hour?
   b. Two consecutive even integers equal 42. Find the integers.

**Teaching Notes for solving algebraic equations:**
- Make sure to familiarize all students with basic mathematical terms and the proper way to translate to algebraic terms.
- First, read the problem carefully and assign a variable for one of the unknown quantities.
- Write expressions if necessary for any other unknown quantities in terms of same variable.
- Write an equation for the stated problem.
- Solve the equation and answer the question.
- Check the solution in the original stated problem.

Answers: 1. a. $3x - 11$
b. $7x + 2$
c. $x + 8$

2. a. $2x - 5 = 11$
$x = 8$
b. $5(x + 8) = 30$
$x = -2$

3. a. $x$ = Joe
$x + 5$ = Bill
$x + (x + 5) = 21$
$x = \$8$ (Joe)
$x + 5 = \$13$ (Bill)

b. $x = 1^{st}$ even integer
$x + 2 = 2^{nd}$ even integer
$x + (x + 2) = 42$
$x = 20$
$x + 2 = 22$

**Mini Lecture 2.6**
Problem Solving in Geometry

**Learning Objectives:**

1. Solve problems using formulas for perimeter and area.
2. Solve problems using formulas for a circle's area and circumference.
3. Solve problems using formulas for volume.
4. Solve problems involving the angles of a triangle.
5. Solve problems involving complementary and supplementary angles.

**Examples:**

1. A triangular flower bed has an area of 48 square feet and a height of 12 feet. Find the base of the flower bed.

2. The diameter of a fish pond is 6 feet. Find the area and circumference of the fish pond. First express answer in terms of $\pi$, then round both answers to the nearest square foot and foot respectively.

3. Which is the better buy: a 3 liter bottle of soft drink for $2.99 or a 1.2 liter bottle for $1.10?

4. Find the volume of a cylinder with a radius of 2 inches and height of 6 inches. Give answer in $\pi$ form and then round answer to nearest cubic inch.

5. A volleyball has a radius of 3 inches. Find how much air is needed to fill the ball. Give answer in $\pi$ form and then round answer to nearest cubic inch.

6. Given a right triangle and knowing that the two acute angles are complementary, find the measure of each if one angle is twice the measure of the other.

**Teaching Notes:**

- Make sure to emphasize the formulas outlined in the section.
- Write formula, substitute the given values, and solve for the unknown.

Answers: 1. base = 8 ft. 2. area = $9\pi$ ft$^2$, 28 ft$^2$; circumference = $6\pi$ ft., 19 ft. 3. 1.2 liter bottle
4. $24\pi$ in$^3$, 75 ft$^3$ 5. $36\pi$ in$^3$, 113 in$^3$ 6. 30°, 60°

## Mini Lecture 2.7
Solving Linear Inequalities

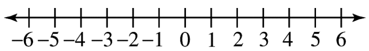
**Learning Objectives:**

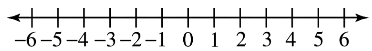
1. Graph the solutions of an inequality on a number line.
2. Use interval notation.
3. Understand properties used to solve linear inequalities.
4. Solve linear inequalities.
5. Identify inequalities with no solution or true for all real numbers.
6. Solve problems using linear inequalities.

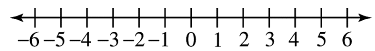
**Examples:**

1. Graph each inequality on the number line.

a. $x \geq -4$ b. $x < 3$ c. $-1 \leq x < 5$








2. Solve each inequality. Write answers in interval notation.

a. $4x - 3 \leq 5$ b. $6 - x \geq 3$ c. $6x - 12 < 8x - 14$

3. Solve each inequality and give the solution in interval notation: Graph solution on a number line.

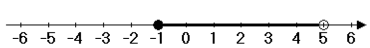
a. $\frac{1}{5}x > -3$ b. $4(6 - 2x) \geq 12 - 4x$ c. $12x - 3 \geq 4(3x + 2)$

d. $5(x - 3) \geq 5x - 15$ e. $20 < 3x + 5$ f. $2(x - 5) > 5x + 3$

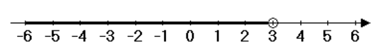
**Teaching Notes:**

- When graphing the solution of an inequality:
  Use a bracket or solid dot when the end point is included in the solution. ($\geq$ or $\leq$)
- When graphing the solution of an inequality:
  Use a parenthesis or open dot when the end point is not included in the solution. ($>$ or $<$)
- When an inequality is multiplied or divided by a negative value, the inequality symbol must be reversed.

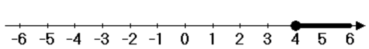
Answers: 1. a.



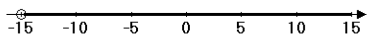

b.

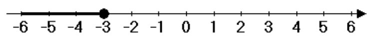


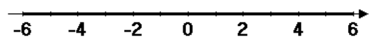

c.

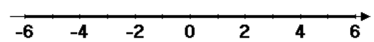


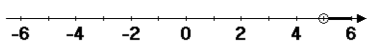

2. a. $(-\infty, 2]$ b. $(-\infty, 3]$ c. $(1, \infty)$

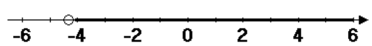
3. a. $x > -15$ $\{x \mid x > -15\}$ 


b. $x \leq 3$ $\{x \mid x \leq 3\}$ 


c. No Solution { } or Ø 


d. All Real Numbers $\{x \mid x \text{ is a real number}\}$ 


e. $x > 5$ $\{x \mid x > 5\}$ 


f. $x > -\dfrac{13}{3}$ $\left\{x \mid x > -\dfrac{13}{3}\right\}$ 

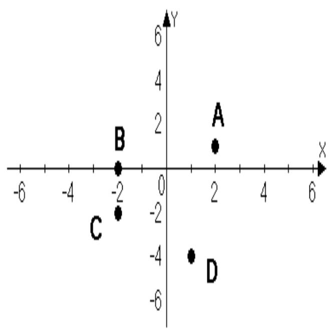
**Mini Lecture 3.1**
Graphing Linear Equations in Two Variables

**Learning Objectives:**

1. Plot ordered pairs in the rectangular coordinate system.
2. Find coordinates of points in the rectangular coordinate system.
3. Determine whether an ordered pair is a solution of an equation.
4. Find solutions of an equation in two variables.
5. Use point plotting to graph linear equations.
6. Use graphs of linear equations to solve problems.

**Examples:**

1. Plot the given points in a rectangular coordinate system. Indicate in which quadrant each point lies.
   a. $(-2, -4)$
   b. $(3, 1)$
   c. $(-2, 3)$
   d. $(5, -2)$
2. Give the ordered pairs that correspond to the points labeled.




3. Determine if the ordered pair is a solution for the given equation.
   a. $2x + 3y = 10$ $(2, 2)$ b. $3x - y = 5$ $(-1, 2)$

4. Find five solutions for $y = 2x - 1$ by completing the table of values.
   a.

| $x$ | $y = 2x - 1$ | $(x, y)$ |
|---|---|---|
| –2 | | |
| –1 | | |
| 0 | | |
| 1 | | |
| 2 | | |

   b. Plot the ordered pairs to graph the line $y = 2x - 1$.

5. Find five solutions for $y = -x + 1$ by completing the table of values.

a.

| $x$ | $y = -x + 1$ | $(x, y)$ |
|---|---|---|
| –2 | | |
| –1 | | |
| 0 | | |
| 1 | | |
| 2 | | |

b. Plot the ordered pairs to graph the quadratic equation $y = -x + 1$.

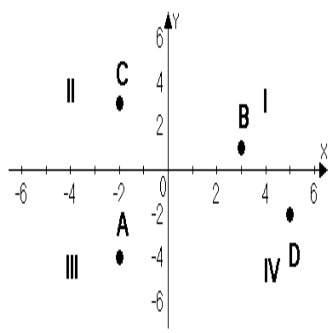
6. a. Your cell phone contract has a base charge of \$10 per month and a \$.03 per minute charge for nation-wide calling. Create a table of values for $y = .03x + 10$. Use 0, 60, 120, 180, 240 for $x$.

b. Plot the ordered pairs to graph the above equation.

**Teaching Notes:**

- When graphing linear equations $(y = mx + b)$ on a coordinate plane, the ordered pairs will form a line when connected.
- It is very important to be able to plot points accurately. Students often have problems with points in the form (0, b).

Answers: 1.

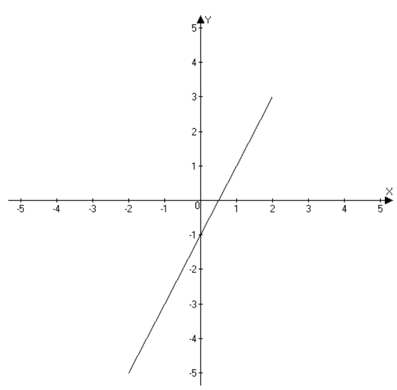



2.a. (2, 1) b. (–2, 0) c. (–1, –2) d. (1, –4)

3.a. yes b. no

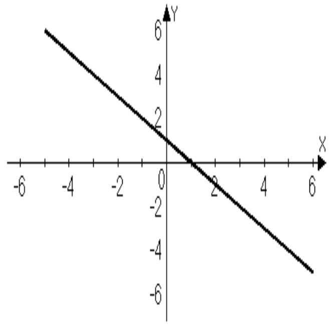
4. a. (–2, –5) (–1, –3) (0, –1) (1, 1) (2, 3)

b.



5. a. (–2, 3) (–1, 2) (0, 1) (1, 0) (2, –1)

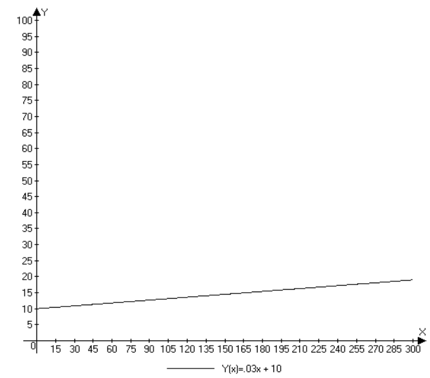
b.




6. a.

| $x$ | $y = 0.03x + 10$ | $(x, y)$ |
|---|---|---|
| 0 | $y = 0 + 10$ | (0, 10) |
| 60 | $y = 1.8 + 10$ | (60, 11.80) |
| 120 | $y = 3.6 + 10$ | (120, 13.60) |
| 180 | $y = 5.4 + 10$ | (180, 15.40) |
| 240 | $y = 7.2 + 10$ | (240, 17.20) |

b.

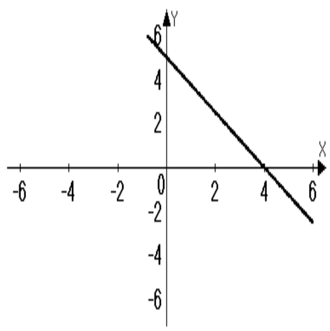


**Mini Lecture 3.2**
Graphing Linear Equations Using Intercepts

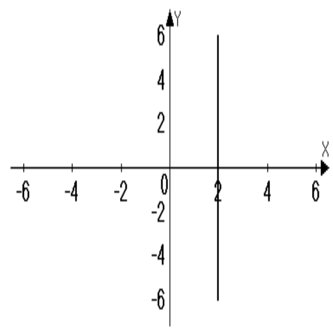
**Learning Objectives:**
1. Use a graph to identify intercepts.
2. Graph a linear equation in two variables using intercepts.
3. Graph horizontal and vertical lines.

**Examples:**
1. Identify the $x$ and $y$-intercepts of each line.

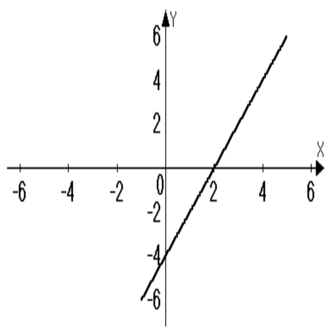
a.




b.




c.




2. Find the $x$-intercept of the graphs of each of the following equations by substituting 0 in for $y$ and solving for $x$.
   a. $4x + 7y = 12$     b. $y = 3x - 3$     c. $x - 2y = -8$

3. Find the $y$-intercept of the graphs of each of the following equations by substituting 0 in for the $x$ and solving for $y$.
   a. $3x + 2y = -12$     b. $y = 2x + 7$     c. $5x - y = 3$

4. Graph each of the following equations by finding the $x$ and $y$-intercepts and a check point. Label the intercepts.
   a. $2x - 4y = 12$     b. $5x + 3y = -15$     c. $y = 2x + 6$

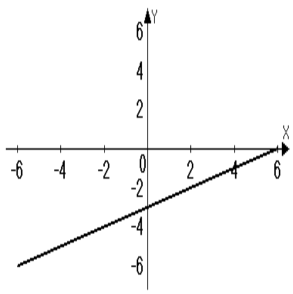
5. Graph each equation on the coordinate plane.
   a. $y + 8 = 12$     b. $x = -3$

**Teaching Notes:**

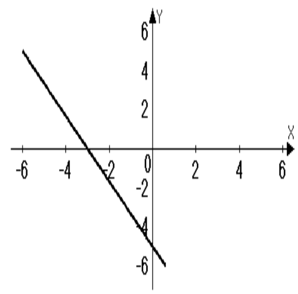
- Intercepts are points not just numbers.
- The $x$-intercept is the point where a graph crosses the $x$ axis. The value of $y$ is always zero at the $x$-intercept.
- The $y$-intercept is the point where a graph crosses the $y$ axis. The value of $x$ is always zero at the $y$-intercept.
- When an equation is in standard form and $a$ and $b$ are factors of $c$, then finding intercepts is a good method to choose for graphing.
- A table is often useful to find intercepts.
- A vertical line has no $y$-intercept, unless it is the $y$-axis ($x = 0$).
- A horizontal line has no $x$-intercept, unless it is the $x$-axis ($y = 0$).

Answers: 1. a. $x$-intercept (4,0); $y$-intercept (0,5) b. $x$-intercept (2,0); $y$-intercept (none) c. $x$-intercept (2,0); $y$-intercept (0, –4) 2. a. (3,0) b. (1,0) c. ( –8,0) 3. a. (0, –6) b. (0,7) c. (0, –3)

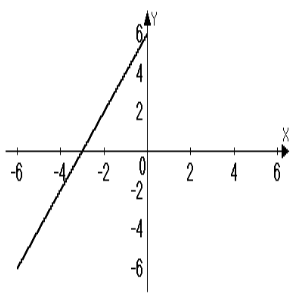
4. a.



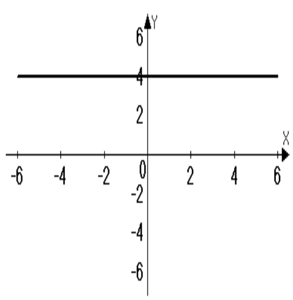
b.



c.



5. a.



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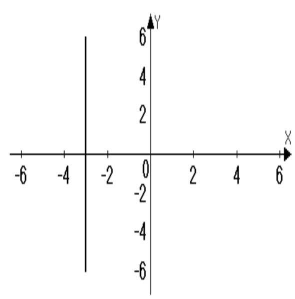


**Mini Lecture 3.3**
Slope

**Learning Objectives:**

1. Compute a line's slope.
2. Use slope to show that lines are parallel.
3. Use slope to show that lines are perpendicular.
4. Calculate rate of change in applied situations.

**Examples:**

1. Using the formula for slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, find the slope of the line passing through each pair of points.
   a. (2, 4) (–3, 1)

   b. (–4, 2) (3, –1)

   c. (1, 5) (2, 5)

   d. (–8, 3) (–8, 1)

2. Determine whether a line passing through points (1, 4) and (5, 6) is parallel or perpendicular to a line passing through points (1, –5) and (0, –3).

3. Determine whether a line passing through points (3, 4) and (5, 2) is parallel or perpendicular to a line passing through points (–3, 5) and (–1, 3).

4. Determine whether a line passing through points (5, –8) and (3, 2) is parallel or perpendicular to a line passing through the points (–6, 3) and (–1, 4).

5 Property taxes have continued to increase year after year. Given that in 1990 a home's taxes were \$1200 and that same home's taxes were \$2600 in 2010. If *x* represents the year and *y* the real estate tax, calculate the slope and explain the meaning of your answer.

**Teaching Notes:**

- Slope is defined as $\dfrac{rise}{run}\left(\dfrac{horizontal\ change}{vertical\ change}\right)$.
- $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where *m* represents slope and comes from the French verb "*monter*" meaning to rise or ascend.
- Four slope possibilities:
  1. $m > 0$, positive slope, rises from left to right
  2. $m < 0$, negative slope, falls from left to right
  3. $m = 0$, line is horizontal
  4. $m$ is undefined, line is vertical

Answers: 1.a. $\frac{3}{5}$ b. $-\frac{3}{7}$ c. 0 d. undefined 2. Perpendicular 3. Parallel 4. Perpendicular

5. slope is $\frac{70}{1}$; taxes went up \$70 per year.

## Mini Lecture 3.4

The Slope-Intercept Form of the Equation of a Line

**Learning Objectives:**

1. Find a line's slope and $y$-intercept of a line from its equation.
2. Graph lines in slope-intercept form.
3. Use slope and $y$-intercept to graph $Ax + By = C$.
4. Use slope and $y$-intercept to model data.

**Examples:**

1. Find the slope and $y$-intercept of each line with the following equations: (Write the $y$-intercept as a point.)

   a. $y = \frac{2}{3}x - 4$ b. $y = -3x + 2$ c. $y = -1$

   d. $y = \frac{1}{2}x$ e. $y = 4x - 5$ f. $y = -\frac{3}{4}x + 8$

2. Put each equation in slope-intercept form by solving for $y$. (Isolate $y$) Then name the slope and $y$-intercept.

   a. $2x + y = -6$ b. $-4x - 3y = 6$ c. $x - 2y = 8$

   d. $5y = 10x + 4$ e. $x + y = 10$ f. $3x - 4y = 7$

3. Graph each equation using the slope and the $y$-intercept.

   a. $4x - 2y = 6$ b. $y = -\frac{1}{2}x + 2$ c. $3x - y = -3$

**Teaching Notes:**

- In an equation in the form $y = mx + b$, $m$ is the slope of line and $b$ is the $y$-coordinate of the $y$-intercept.
- To graph: use the $y$-intercept as the starting point. Then use the slope to plot at least two more points.
- Remember, the slope must be in fraction form, $\frac{rise}{run}$. If the slope is an integer, it can be put over 1 to form a fraction.

Answers: 1. a. slope $\frac{2}{3}$; $y$-intercept $(0, -4)$ b. slope $-3$; $y$-intercept $(0, 2)$ c. slope 0; $y$-intercept $(0, -1)$

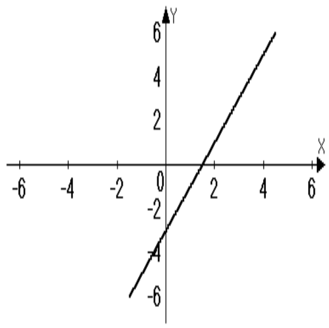
d. slope $\frac{1}{2}$; $y$-intercept $(0, 0)$ e. slope 4, $y$-intercept $(0, -5)$ f. slope $-\frac{3}{4}$; $y$-intercept $(0, 8)$

2. a. $y = -2x - 6$; slope $-2$; $y$-intercept $(0, -6)$ b. $y = -\frac{4}{3}x - 2$; slope $-\frac{4}{3}$; $y$-intercept $(0, -2)$

c. $y = \frac{1}{2}x - 4$; slope $\frac{1}{2}$; $y$-intercept $(0, -4)$ d. $y = 2x + \frac{4}{5}$; slope 2; $y$-intercept $(0, \frac{4}{5})$

e. $y = -x + 10$; slope $-1$, $y$-intercept $(0, 10)$ f. $y = \frac{3}{4}x - \frac{7}{4}$; slope $\frac{3}{4}$, $y$-intercept $\left(0, -\frac{7}{4}\right)$

3. a.

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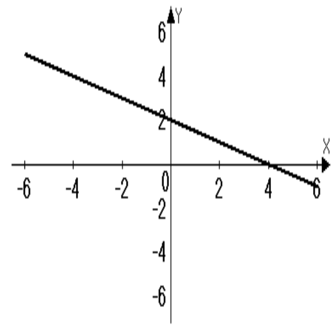
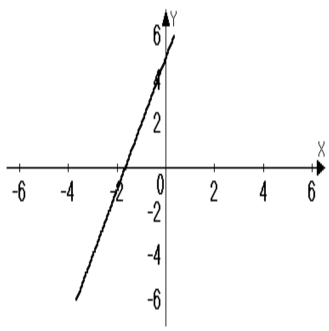



c.



**Mini Lecture 3.5**
The Point-Slope Form of the Equation of a Line

**Learning Objectives:**

1. Use the point-slope form to write equations of a line.
2. Write linear equations that model data and make predictions.

**Examples:**

1. Write the point-slope form and the slope-intercept form of the equation of the line with slope 3 that passes through the point (–1, 4).
2. Write the point-slope form and the slope-intercept form of the equation of the line through the points (1, 4) (–2, 3).
3. The cost of graphing calculators over time has decreased. In 2000, one particular brand sold for \$136, in 2011 that same calculator sold for \$92. Use the coordinates of the two points (2000, 136) (2011, 92) to write the equation in point-slope and slope-intercept form.

**Teaching Notes:**

- Distinguish between 2 forms of an equation of a line: point-slope (4.3) and slope-intercept (4.4).
- Point-slope equation with the slope of $m$ and passing through point $(x_1, y_1)$ is $y - y_1 = m(x - x_1)$.
- Use the point-slope equation when given at least one point and the slope.
- If given two points and asked to find the equation of the line, one must first find the slope, section 3.3 and then use the slope and one of the given points in the point-slope equation.

Answers: 1. $y = 4(x + 1)$; $y = 3x + 7$ 2. $y - 4 = \frac{1}{3}(x - 1)$; $y = \frac{1}{3}x + \frac{11}{3}$
3. $y - 136 = -4(x - 2000)$; $y = -4x + 8136$ or $y - 92 = -4(x - 2011)$; $y = -4x + 8136$

**Mini Lecture 3.6**
Linear Inequalities in Two Variables

**Learning Objectives:**
1. Determine whether an ordered pair is a solution of an inequality.
2. Graph linear inequalities in two variables.
3. Solve applied problems involving linear inequalities in two variables.

**Examples:**
Determine which ordered pairs are solutions to the given inequalities.

1. $2x + 3y < 10$
   a. $(-1, 4)$ b. $(4, -1)$ c. $(0, 3)$ d. $(3, 2)$

2. $y \geq -x + 3$
   a. $(4, 7)$ b. $(-3, 0)$ c. $(5, 2)$ d. $(-1, -1)$

3. $4x - 2y \leq 8$
   a. $(0, 2)$ b. $(2, 0)$ c. $(-2, -2)$ d. $(1, -5)$

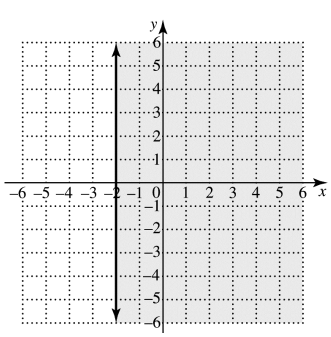
Put each inequality in slope-intercept form then graph the boundary line and shade the appropriate half plane.

4. a. $2x + 7 \geq 3$ b. $3x - 3y > 6$ c. $y \geq \frac{1}{4}x - 3$
   d. $x < y$ e. $4x + 6y \leq -12$ f. $x \geq 4$

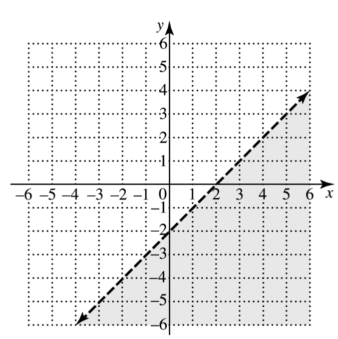
**Teaching Notes:**
- An inequality has many solutions, therefore shading must be used to show the solutions.
- The graphed line is a boundary and it divides the coordinate plane into two half planes, one of which will be shaded.
- Remind students to use a solid line if the points on the line are included in the solution ($\geq$ or $\leq$).
- Remind students to use a broken or dashed line if points on the line are not included in the solution ($>$ or $<$).
- Students tend to forget to reverse the inequality symbol when multiplying or dividing both sides by a negative number.
- Remind students to use a straight edge when graphing lines.

Answers: 1.no; yes; yes; no 2. yes; no; yes; no 3. yes; yes; yes; no

4. a.

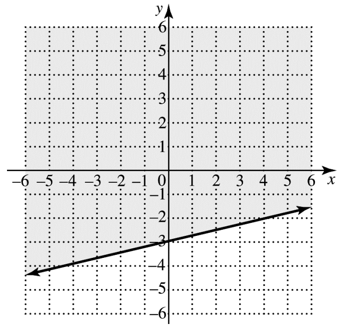


b.

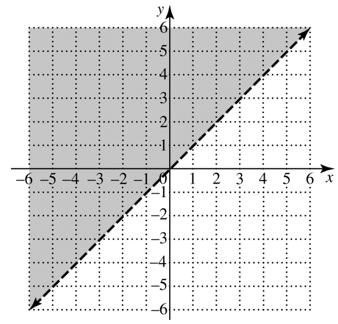


Boundary line dashed

c.

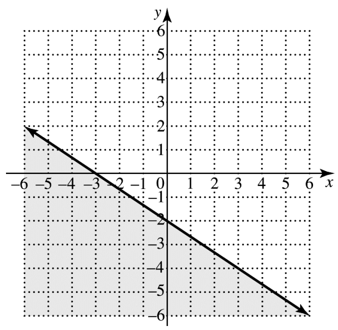


d.

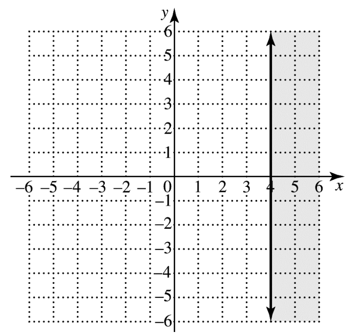


Boundary line dashed

e.



f.



## Mini Lecture 4.1

Solving Systems of Linear Equations by Graphing

**Learning Objectives:**

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve systems of linear equations by graphing.
3. Use graphing to identify systems with no solution or infinitely many solutions.
4. Use graphs of linear systems to solve problems.

**Examples:**

1. Consider the system.

   $x + y = -3$
   $2x + y = 1$

   Determine if each ordered pair is a solution of the system.

   a. (4, 7)  b. (4, –7)

2. Solve the following systems by graphing. State the solution (the intersection point) as an ordered pair (x, y) or state if there is no solution, or state if there are an infinite number of solutions.

   a. $2x + y = -3$
   $y = -2x - 3$

   b. $2x + y = 3$
   $3x - 2y = 8$

   c. $x + 2y = 6$
   $x + 2y = 2$

**Teaching Notes:**

- When graphing a system of linear equations, there are three possible outcomes:
  1. The two lines can intersect at one point, meaning there is one solution to the system.
  2. The two lines can be parallel to one another, meaning there is no solution to the system.
  3. The two lines are identical or coincide, meaning there are infinitely many solutions to the system.
- When two lines are parallel the system is inconsistent and has no solution.
- When two lines are coinciding, they are called dependent equations and have infinitely many solutions.

Answers:

1. a. not a solution b. yes, a solution 2. a. infinitely many solutions b. (2, –1) c. lines parallel, no solution

## Mini Lecture 4.2

Solving Systems of Linear Equations by the Substitution Method

**Learning Objectives:**

1. Solve linear systems by the substitution method.
2. Use the substitution method to identify systems with no solution or infinitely many solutions.
3. Solve problems using the substitution method.

**Examples:**

Solve each system using the substitution method. If there is no solution or an infinite number of solutions, so state.

1. a. $x + y = 3$, $y = x + 5$  b. $3x - 2y = 5$, $x = 4y - 5$  c. $7x + 6y = -9$, $y = -2x + 1$  d. $5x - 6y = -4$, $x = y$

2. a. $x + 3y = 4$, $x - 2y = -1$  b. $-2x - y = -3$, $3x + y = 0$  c. $8x - y = 15$, $3x + 4y = 10$  d. $3x - 5y = 12$, $x + 2y = 4$

3. a. $3x + 5y = -3$, $x - 5y = -5$  b. $2x - 4y = -4$, $x + 2y = 8$  c. $7x - 6y = -1$, $x - 2y = -1$  d. $2x - y = 1$, $4x + y = 8$

4. a. $6x + 3y = 1$, $y = -2x - 5$  b. $4x - 4y = 8$, $x - y = 2$  c. $4x - 2y = 8$, $2x - y = 4$  d. $y = -3x + 2$, $6x + 2y = 1$

**Teaching Notes:**

- Students like to follow specific steps so give them a list of steps to use for solving systems by substitution. Begin with: Isolate a variable with a coefficient of 1 first.
- Many students think they must solve for *y*. Stress that it does not matter whether the variable solved for is *x* or *y*.
- Use colored pens or markers to underline in one equation what will be substituted in the other equation.
- If a graphing calculator is being used in the class, graphing on the calculator is a good way to check solutions.

Answers: 1. a. $(-1, 4)$ b. $(3, 2)$ c. $(3, -5)$ D. $(4, 4)$ 2. a. $(1, 1)$ b. $(-3, 9)$ c. $(2, 1)$ d. $(4, 0)$ 3. a. $(-2, \frac{3}{5})$ b. $(3, \frac{5}{2})$ c. $(\frac{1}{2}, \frac{3}{4})$ d. $(\frac{3}{2}, 2)$ 4. a. No solution b. Infinite solutions c. Infinite solutions d. No solution

**Mini Lecture 4.3**

Solving Systems of Linear Equations by the Addition Method

**Learning Objectives:**

1. Solve linear systems by the addition method.
2. Use the addition method to identify systems with no solution or infinitely many solutions.
3. Determine the most efficient method for solving a linear system.

**Examples:**

Solve the following systems by the addition method.

1. $x + y = 10$
   $x - y = 8$
2. $4x + 3y = 7$
   $-4x + y = 5$
3. $3x - y = 8$
   $x + 2y = 5$
4. $2w - 3z = -1$
   $3w + 4z = 24$
5. $4x - 5y = 8$
   $-4x + 5y = -8$
6. $2x = 5y + 4$
   $2x - 5y = 6$

**Teaching Notes:**

- When solving a system of linear equations there are three methods:
  Graphing (4.1)
  Substitution (4.2)
  Addition (4.3)
- Any of the three methods will work when solving a system and produce the correct answer.
- Teach students how to determine which of the three methods is the most efficient when solving a system of equations.

Answers: 1. (9, 1) 2. $\left(-\frac{1}{2}, 3\right)$ 3. (3, 1) 4. (4, 3) 5. infinitely many solutions 6. no solution

**Mini Lecture 4.4**
Problem Solving Using Systems of Equations

**Learning Objectives:**

1. Solve problems using linear systems.

**Examples:**

Use variables to represent unknown quantities. Write a: Let $x$ = and $y$ = statement for each problem. (Do not solve).

1. The sum of two numbers is 14. One number is six times larger than the other. Find the two numbers.
2. Three pairs of socks and two pairs of mitten cost $42. One pair of the same kind of socks and four pair of the mittens cost $24. Find out how much one pair of socks and one pair of mittens cost.
3. John has $5 bills and $10 bills in his wallet. He has a total of $80. He has twice as many $5 bills as $10 bills. How many $5 bills and how many $10 bills does he have?

Now, for problems 4 – 6, write a system of equations that models the conditions of each problem. (Do not solve).

4.
5.
6.

Solve each of the following using a system of equations.

7. The sum of two numbers is 11. The second number is 1 less than twice the first number. Find the two numbers.
8. Alexis has $1.65 in nickels and quarters. She has 9 coins altogether. How many coins of each kind does she have?
9. Paul invested $12,000 in two accounts. One account paid 4% interest and one account paid 5% interest. At the end of the year his money had earned $560 in interest. How much did he invest in each account?
10. A department store receives 2 shipments of bud vases and picture frames. The first shipment of 5 bud vases and 4 picture frames costs $62. The second shipment of 10 bud vases and 3 picture frames cost $84. Find the cost of a vase and a picture frame.

**Teaching Notes:**

- Stress the importance of reading the problem several times before beginning. Reading aloud really helps.
- Have students write a Let $x$= and $y$ = statements for each word problem before trying to write the system of equations.
- Help students look at the system they have created and determine which method of solving will work best.
- Remind students to make sure their answers make sense for the given situation.
- Try to build confidence with word problems.

Answers: 1. Let $x$ = one number; let $y$ = the other number. 2. Let $x$ = cost of 1 pair of socks; let $y$ = cost of 1 pair of mittens. 3. Let $x$ = number of $5 bills; let $y$ = number of $10 bills

4. $x + y = 14$, $x = 6y$ 5. $3x + 2y = 42$, $x + 4y = 24$ 6. $5x + 10y = 80$, $x = 2y$

7. The numbers are 4 and 7 8. 3 nickels, 6 quarters 9. $4000 invested @ 4% and $8000 invested @ 5%
10. bud vases $6, picture frames $8

**Mini Lecture 4.5**
Systems of Linear Inequalities

**Learning Objectives:**

1. Use mathematical models involving systems of linear inequalities.
2. Graph the solution sets of systems of linear inequalities.

**Examples:**

1. Graph the solution set of each system.

   a. $y < -x + 3$
   $y \geq x - 4$

   b. $y > -\frac{1}{2}x + 3$
   $x \geq 3$

   c. $y \geq x - 1$
   $y < -\frac{1}{3}x + 1$

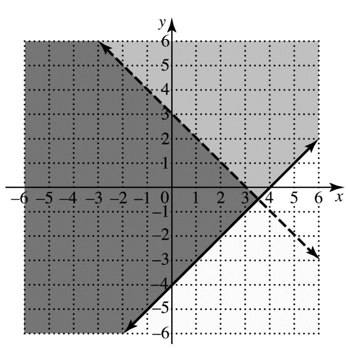
2. Name one point that is a solution for each system of linear inequalities in examples 1a, 1b, and 1c.
   a. b. c.

**Teaching Notes:**

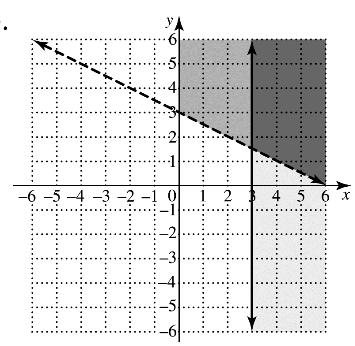
- When the inequality symbol is > or <, the line should be dashed (- - - - -).
- When the inequality symbol is ≥ or ≤, the line should be solid ( ________).
- When graphing inequalities, it is easy to see the overlap of the graphs if different colored pencils are used to graph each inequality.

Answers:

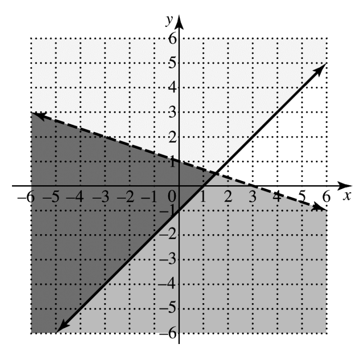
1. a.



b.



c.



2. a. Answers will vary b. Answers will vary c. Answers will vary

## Mini Lecture 5.1
Adding and Subtracting Polynomials

**Learning Objectives:**
1. Understand the vocabulary used to describe polynomials.
2. Add polynomials.
3. Subtract polynomials.
4. Graph equations defined by polynomials of degree 2.

**Examples:**

1. Identify each polynomial as a monomial, a binomial, or a trinomial. Give the degree of the polynomial.
   a. $5x-1$
   b. $9x^2$
   c. $8$
   d. $3x^2-2x+1$

2. Add: $(5x^3+3x^2-5x+4)+(-2x^3+4x^2-8x-2)$

3. Add:
$$\begin{array}{r} -9x^3+4x^2-5x+3 \\ \underline{2x^3 \qquad\quad +3x-7} \end{array}$$

4. Subtract: $(4x^4+3x^3+2x-7)-(-2x^3+x^2-4x+5)$

5. Subtract: $3x^2+5x+4$ from $8x^2-2x-1$

6. Subtract:
$$\begin{array}{r} 10x^4+3x^3-4x^2 \qquad +5 \\ -\underline{(\qquad 2x^3+3x^2+4x \qquad)} \end{array}$$

7. Find seven solutions for $y=x^2+2$ by completing the table of values.
   a.

| $x$ | $y=x^2+2$ | y |
|---|---|---|
| –3 | | |
| –2 | | |
| –1 | | |
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |

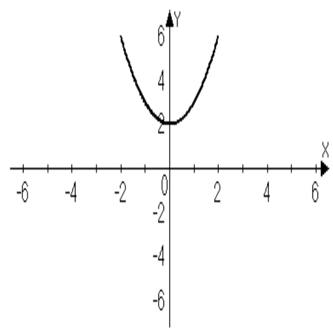
   b. Plot the ordered pairs to graph the quadratic equation $y=x^2+2$.

**Teaching Notes:**

- Vocabulary terms are very important when teaching/learning about polynomials. Example: monomial, degree of polynomial, coefficient, exponent.
- When adding or subtracting polynomials either the horizontal or vertical format may be used.
- When using the vertical format, if a particular term is missing, leave a vacant space making sure to line up like terms in columns.
- When subtracting, make sure to change the sign of each term being subtracted to its opposite, then use rules for adding like and unlike signs.

Answers: 1. a. binomial, 2 b. monomial, 2 c. monomial, 0 d. trinomial, 2 2. $3x^3+7x^2-13x+2$
3. $-7x^3+4x^2-2x-4$ 4. $4x^4+5x^3-x^2+6x-12$ 5. $5x^2-7x-5$
6. $10x^4+x^3-7x^2-4x+5$
7.

| ***x*** | $y=x^2+2$ | **y** |
|---|---|---|
| $-3$ | $y=(-3)^2+2$ | |
| $-2$ | $y=(-2)^2+2$ | |
| $-1$ | $y=(-1)^2+2$ | |
| 0 | $y=(0)^2+2$ | |
| 1 | $y=(1)^2+2$ | |
| 2 | $y=(2)^2+2$ | |
| 3 | $y=(3)^2+2$ | |



## Mini Lecture 5.2
Multiplying Polynomials

**Learning Objectives:**
1. Use the product rule for exponents.
2. Use the power rule for exponents.
3. Use the products-to-powers rule for exponents.
4. Multiply monomials.
5. Multiply a monomial and a polynomial.
6. Multiply polynomials when neither is a monomial.

**Examples:**
Simplify each expression using the exponent rule.

1. a. $x^5 \cdot x^2$  b. $a^2 \cdot a \cdot a^8$  c. $4^2 \cdot 4^4$  d. $y \cdot y \cdot y$
2. a. $(x^4)^2$  b. $(a^5)^5$  c. $(4^2)^4$  d. $(y^5)^2$
3. a. $(3a)^4$  b. $(6x^3)^2$  c. $(2a^2b^3c^5)^3$  d. $(-6x^3)^4$
4. $4y^2 \cdot 3y^4$  b. $(-3x)(-2x^3)$  c. $(8x^2y^2)(3x^2y^4)$  d. $(5x^3)(-3x^3y^7)$
5. a. $5a(a^2 + 2)$  b. $3x(2x - 4)$  c. $5y^2(y^2 - y + 2)$  d. $2x^2y(5x^3y - 2xy^3)$
6. a. $(a + 2)(a + 8)$  b. $(2x - 3)(3x - 2)$  c. $(y + 7)(2y - 3)$  d. $(2b - 5)(5b - 2)$
7. a. $(x + 2)(x^2 + 3x + 4)$  b. $(a + 4)(a^2 - 3a + 1)$
   c. $(y - 3)(2y^2 + 3y + 5)$  d. $(x - 2)(2x^3 - 4x^2 + 3x - 6)$

**Teaching Notes:**
- Have students write out exponent rules in words on one (1) page with examples for referral and study.
- Students find each rule easy as presented but the rules get jumbled when used together.
- Practice often and review.
- A very common mistake will be students multiplying bases if they are numbers – warn against this! The base stays the same when multiplying like bases.

Answers: 1. a. $x^7$ b. $a^{11}$ c. $4^6$ d. $y^3$ 2. a. $x^8$ b. $a^{25}$ c. $4^8$ d. $y^{10}$ 3. a. $81a^4$ b. $36x^6$ c. $8a^6b^9c^{15}$ d. $1296x^{12}$ 4. a. $12y^6$ b. $6x^4$ c. $24x^4y^6$ d. $-15x^6y^7$ 5. a. $5a^3 + 10a$ b. $6x^2 - 12x$ c. $5y^4 - 5y^3 + 10y^2$ d. $10x^5y^2 - 4x^3y^4$ 6. a. $a^2 + 10a + 16$ b. $6x^2 - 13x + 6$ c. $2y^2 + 11y - 21$ d. $10b^2 - 29b + 10$
7. a. $x^3 + 5x^2 + 10x + 8$ b. $a^3 + a^2 - 11a + 4$ c. $2y^3 - 3y^2 - 4y - 15$
d. $2x^4 - 8x^3 + 11x^2 - 12x + 12$

# Mini Lecture 5.3
Special Products

**Learning Objectives:**
1. Use FOIL in polynomial multiplication.
2. Multiply the sum and difference of two terms.
3. Find the square of a binomial sum.
4. Find the square of a binomial difference.

**Examples:**
Use the FOIL method to find each product.

1. $(x+2)(x+4)$

2. $(3x-7)(2x+3)$

3. $(3-5x)(2+4x)$

4. Find each product using the rule for finding the product of the sum and difference of two terms.
   a. $(2y+4)(2y-4)$
   b. $(6x-1)(6x+1)$
   c. $(4x^3+8)(4x^3-8)$

5. Multiply using the rules for the square of a binomial.
   a. $(x+4)^2$
   b. $(3x+7)^2$
   c. $(5x-2)^2$
   d. $(4x+8)^2$

**Teaching Notes:**
- When multiplying two binomials, the FOIL method is often used.

| **F** | **O** | **I** | **L** |
|---|---|---|---|
| product of **first** terms, | product of **outside** terms, | product of **inside** terms, | product of **last** terms |

- When multiplying the sum and difference of two terms, $(A+B)(A-B)$, the outside product and inside product will have a sum of zero resulting in an answer of $A^2 - B^2$.
- When multiplying the square of a binomial sum $(A+B)^2$, the outside product and inside product will be identical resulting in $A^2 + AB + AB + B^2 = A^2 + 2AB + B^2$.

Answers: 1. $x^2+6x+8$ 2. $6x^2-5x-21$ 3. $6+2x-20x^2$ 4. a. $4y^2-16$ b. $36x^2-1$ c. $16x^6-64$ 5. a. $x^2+8x+16$ b. $9x^2+42x+49$ c. $25x^2-20x+4$ d. $16x^2+64x+64$

## Mini Lecture 5.4
Polynomials in Several Variables

**Learning Objectives:**
1. Evaluate polynomials in several variables.
2. Understand the vocabulary of polynomials in two variables.
3. Add and subtract polynomials in several variables.
4. Multiply polynomials in several variables.

**Examples:**

Evaluate the following polynomial given values for the variables:

1. $2x^2 - xy + 3y^2$ a. $x = 2 \quad y = -1$ b. $x = -2 \quad y = -2$

2. $5x^3 + 2x^2y - 3xy - y^2$ a. $x = 1 \quad y = 3$ b. $x = -2 \quad y = -1$

Answer the following questions about the following polynomials:

3. $8x^4y^3 - 5xy^2 + 4x^3y - x^2y^6$ 4. $3x^5y - x^4y^3 + 2x^2y^2 + xy^3 + 4y^4$

   a. Name the coefficient of each term.
   b. Tell the degree of each term.
   c. What is the degree of the polynomial?

5. Add or subtract the polynomials.
   a. $(6x^2y + 2xy - 3xy^2) + (4x^2y - xy + 5xy^2)$
   b. $(10x^3y^2 - 5x^2y + 2xy^2 - 3xy^3) - (4x^3y^2 - 5x^2y + 4xy^2 - 6xy)$
   c. $(3a^4b^2 + 2a^3b - 5a^2b^3) + (5a^4b^2 + 3ab^3 + 4a^2b^3)$
   d. $(5x^4y^3 + 3x^2y^2 + 2xy^4) - (3x^4y^4 - 3x^2y^2 - 4xy^4)$

6. Multiply the polynomials.
   a. $(4x^2y^2)(7x^3y^3)$ b. $(-2xy^3)(5x^3y)$
   c. $5x^2y^2(3x^3y^2 - 4xy^3 + 6x^2y)$ d. $(2x + 3y)(x - 4y)$
   e. $(4a + 3b)(4a + b)$ f. $(3x + 5y)^2$
   g. $(2x - 8y)^2$ h. $(x - y)(2x^2 - 3xy + y^2)$

**Teaching Notes:**
- It is very important to use parentheses when substituting for a variable.
- Students need to be reminded that the sign in front of a term is the sign of the coefficient of the term.
- Students will have problems with the degree of the polynomial. Remind them it is the same as the one term with the highest degree. DO NOT add all the exponents in the polynomial.

Answers: 1. a. 13 b. 16 2. a. –7 b. –55 3. a. 8, –5, 4, –1 b. 7, 3, 4, 8 c. degree 8
4. a. 3, –1, 2, 1, 4 b. 6, 7, 4, 4, 4 c. degree 7 5. a. $10x^2y + xy + 2xy^2$ b. $6x^3y^2 - 2xy^2 + 3xy^3$
c. $8a^4b^2 + 2a^3b + 3ab^3 - a^2b^3$ d. $2x^4y^3 + 6x^2y^2 + 6xy^4$ 6. a. $28x^5y^5$ b. $-10x^4y^4$
c. $15x^5y^4 - 20x^3y^5 + 30x^4y^3$ d. $2x^2 - 5xy - 12y^2$ e. $16a^2 + 16ab + 3b^2$
f. $9x^2 + 30xy + 25y^2$ g. $4x^2 - 32xy + 64y^2$ h. $2x^3 - 5x^2 + 4xy^2 - y^3$

**Mini Lecture 5.5**
Dividing Polynomials

**Learning Objectives:**

1. Use the quotient rule for exponents.
2. Use the zero–exponent rule.
3. Use the quotients-to-powers rule.
4. Divide monomials.
5. Check polynomial division.
6. Divide a polynomial by a monomial.

**Examples:**

1. Divide each expression using the quotient rule:

   a. $\dfrac{7^5}{7^2}$ b. $\dfrac{x^{11}}{x^7}$ c. $\dfrac{y^8}{y}$

2. Use the zero-exponent rule to simplify each expression.

   a. $8^0$ b. $(-8)^0$ c. $-8^0$ d. $30x^0$ e. $(30x)^0$

3. Simplify each expression using the quotients-to-powers rule.

   a. $\left(\dfrac{a}{6}\right)^2$ b. $\left(\dfrac{x^4}{3}\right)^3$ c. $\left(\dfrac{2y^3}{z^4}\right)^4$

4. Divide the monomials.

   a. $\dfrac{-24x^8}{6x^4}$ b. $\dfrac{2x^6}{6x^6}$ c. $\dfrac{12x^6y^4}{2x^3y}$

5. Find the quotient.

   a. $\dfrac{-16x^8+12x^5-2x^2}{2x}$ b. $\dfrac{30x^6-25x^5+10x^3}{5x^3}$ c. $\dfrac{24x^6y^3-6x^4y^2+12x^2y}{6xy}$

**Teaching Notes:**

- When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the denominator. Ex: $\left(\dfrac{a^x}{a^y}\right)=a^{x-y}$ .
- Any nonzero base raised to the 0 power equals 1.
- When dividing monomials, divide the coefficients and divide the variables. When dividing the variables, keep the variable and subtract the exponents.
- To check a division problem, divisor · quotient = dividend.
- When dividing a polynomial a monomial by a monomial, divide each term by the monomial.

Answers: 1.a. $7^3$ b. $x^4$ c. $y^7$ 2.a. 1 b. 1 c. –1 d. 30 e. 1 3.a. $\dfrac{a^2}{36}$ b. $\dfrac{x^{12}}{27}$ c. $\dfrac{16y^{12}}{z^{16}}$

4.a. $-4x^4$ b. $\dfrac{1}{3}$ c. $6x^3y^3$ 5.a . $-8x^7+6x^4-x$ b. $6x^3-5x^2+2$ c. $4x^5y^2-x^3y+2x$

**Mini Lecture 5.6**
Dividing Polynomials by Binomials

**Learning Objectives:**

1. Divide polynomials by binomials.

**Examples:**

Divide and check: (no remainders)

1. a. $\dfrac{x^2+5x+6}{x+3}$ b. $\dfrac{2a^2-9a-5}{2a+1}$ c. $(a^2+9a+20)\div(a+4)$

Divide and check: (some will have remainders)

2. a. $\dfrac{x^2+5x+6}{x+3}$ b. $\dfrac{x^2+5x-6}{x+1}$ c. $(6a^2+5a+1)\div(2a+3)$

d. $\dfrac{6a^3-13a^2-4a+15}{3a-5}$ e. $(2x^4-13x^3+16x^2-9x+20)\div(x-5)$

f. $\dfrac{7x^2-4x+12+3x^3}{x+1}$ g. $(20x^3-8x^2+5x-5)\div(5x-2)$

Divide. Fill in any missing terms.

3. a. $(3x^4-25x^2-20)\div(x-3)$ b. $\dfrac{y^3-8}{y-2}$ c. $\dfrac{3x^2-4}{x-1}$

**Teaching Notes:**

- Dividing polynomials is like long division. Most students can divide using long division. Remind about writing remainders as fractions.
- Teach this concept by comparing to long division.
- Students often find it helpful to cover up the second term of a binomial before dividing.
- Using parentheses before subtracting may help reduce sign errors.
- It is often helpful to write these steps vertically on the page when doing examples with the students:
  Cover up; divide; uncover; multiply back; parentheses; subtract; bring down, cover up; divide, uncover; multiply back; parentheses; subtract; bring down… Students are able to follow the above pattern if they practice enough. Remind students that the divisor and the dividend must be in DESCENDING powers of the variable WITHOUT skipping any powers.
- The sign errors when subtracting will be the biggest obstacle in getting the correct answer.
- Students like to see how they can check their answers.

Answers: 1. a. $x+2$ b. a – 5 c. a + 5 2. a. $x+2+\dfrac{2}{x+3}$ b. $x+4+\dfrac{-10}{x+1}$ c. $3a-2+\dfrac{7}{a+3}$
d. $2a^2-a-3$ e. $2x^3-3x^2+x-4$ f. $3x^2+4x-8+\dfrac{20}{x+1}$ g. $4x^2+1+\dfrac{-3}{5x-2}$
3. a. $3x^3+9x^2+2x+6+\dfrac{-2}{x-3}$ b. $y^2+2y+4$ c. $3x+3+\dfrac{-1}{x-1}$

# Mini Lecture 5.7

Negative Exponents and Scientific Notation

**Learning Objectives:**

1. Use the negative exponent rule.
2. Simplify exponential expressions.
3. Convert from scientific notation to decimal notation.
4. Convert from decimal notation to scientific notation.
5. Compute with scientific notation.
6. Solve applied problems with scientific notation.

**Examples:**

1. Rewrite each expression with a positive exponent, then simplify if possible.

   a. $3^{-2}$  b. $4^{-3}$  c. $(-2)^{-4}$  d. $-2^{-4}$  e. $5^{-1}$

   f. $\dfrac{2^{-2}}{5^{-3}}$  g. $\left(\dfrac{5}{7}\right)^{-2}$  h. $\dfrac{1}{4y^{-3}}$  i. $\dfrac{x^{-3}}{y^{-5}}$

2. Simplify the following, make sure all answers are written with positive exponents.

   a. $x^{-10} \cdot x^{4}$  b. $\dfrac{x^{2}}{x^{9}}$  c. $\dfrac{24x^{3}}{8x^{7}}$

   d. $\dfrac{100y^{4}}{25y^{10}}$  e. $\dfrac{(3x^{3})^{2}}{x^{10}}$  f. $\left(\dfrac{x^{6}}{x^{2}}\right)^{-3}$

3. Write each number in scientific notation.

   a. 3,840,000  b. 0.000158

4. Write each number in decimal notation.

   a. $9.2\times10^{-3}$  b. $3.851\times10^{5}$

5. Perform the indicated computation writing the answers in scientific notation.

   a. $(4\times10^{4})(2\times10^{8})$  b. $\dfrac{4.2\times10^{8}}{2\times10^{-2}}$

6. If $3.29\times10^{5}$ people live in your city, what is the population of your hometown in decimal notation?

**Teaching Notes:**

- To simplify exponential expressions remember: remove all parentheses, no powers are raised to powers, each base occurs only once, no negative or zero exponents appear.
- If the base is any real number other than 0 and n is a negative number, then $x^{-n} = \frac{1}{x^n}$.
- A positive number is written in scientific notation when it is expressed in the form $a \times 10^n$ where *a* is a number between 1 and 10 and *n* is an integer.
- When changing scientific notation to decimal notation, move the decimal to the right "*n*" places if *n* is negative.

Answers: 1. a. $\frac{1}{3^2} = \frac{1}{9}$ b. $\frac{1}{4^3} = \frac{1}{64}$ c. $\frac{1}{(-2)^4} = \frac{1}{16}$ d. $\frac{1}{-2^4} = -\frac{1}{16}$ e. $\frac{1}{5^1} = \frac{1}{5}$

f. $\frac{5^3}{2^2} = \frac{125}{4}$ g. $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$ h. $\frac{y^3}{4}$ i. $\frac{y^5}{x^3}$ 2. a. $\frac{1}{x^6}$ b. $\frac{1}{x^7}$ c. $\frac{3}{x^4}$ d. $\frac{4}{y^6}$ e. $\frac{9}{x^4}$

f. $\frac{1}{x^{12}}$ 3. a. $3.84 \times 10^6$ b. $1.58 \times 10^{-4}$ 4. a. 0.0092 b. 385,100 5. a. $(8 \times 10^{12})$

b. $(2.1 \times 10^{10})$ 6. 329,000 people

## Mini Lecture 6.1
The Greatest Common Factor and Factoring by Grouping

**Learning Objectives:**

1. Find the greatest common factor.
2. Factor out the greatest common factor of a polynomial.
3. Factor out the negative of the greatest common factor of a polynomial.
4. Factor by grouping.

**Examples:**

1. Find the greatest common factor for each list of terms.
   a. $24x^3$ and $12x^2$ b. $-18x^2, 30x^5$, and $48x$ c. $x^2y^4, x^3y$, and $x^5y^7$

2. Factor the following:
   a. $9x^2+27$ b. $45x^2+50x^5$ c. $18x^6-6x^3+24x^2$
   d. $12x^2y^3-8x^3y^2+16xy^4$ e. $10x^3y+15x^2y^2-5xy$

3. Factor out the negative of the greatest common factor of a polynomial.
   a. $-3x^2+36$ b. $-12a^4b^3+6a^2b^2-9a^3b$

4. Factor.
   a. $x^2(x-4)+3(x-4)$ b. $x(x^2+9)+(x^2+9)$

5. Factor by grouping.
   a. $6x^2-10x+9x-15$ b. $2x^4+2x^2-5x^2-5$

**Teaching Notes:**

- To factor means to find an equivalent expression whose product gives the original polynomial.
- The greatest common factor (GCF) is an expression of the highest degree that will divide into each term of a polynomial.
- To factor by grouping: first group terms that have a common monomial factor. Next, factor out the common monomial from each group, and then factor out the remaining common binomial factor (if one exists).
- Factoring can be easily checked by multiplying the terms through distributing or FOIL (if two binomials.

Answers: 1. a. $12x^2$ b. $6x$ c. $x^2y$ 2. a. $9(x^2+3)$ b. $5x^2(9+10x^3)$ c. $6x^2(3x^4-x+4)$ d. $4xy^2(3xy-2x^2+4y^2)$ e. $5xy(2x^2+3xy-1)$ 4. $4xy^2(3xy-2x^2+4y^2)$ e. $5xy(2x^2+3xy-1)$ 3. a. $a-3(x^2-12)$ b. $-3a^2b(4a^2b^2-2b+3a)$ 4. a. $(x-4)(x^2+3)$ b. $(x^2+9)(x+1)$ 5. a. $(3x-5)(2x+3)$ b. $(x^2+1)(2x^2-5)$

**Mini Lecture 6.2**
Factoring Trinomials Whose Leading Coefficient is 1

**Learning Objectives:**

1. Factor trinomials of the form $ax^2 + bx + c$.

**Examples:**

Factor these similar problems. Make an observation of the signs.

1. a. $x^2 + 9x + 18$ b. $x^2 - 9x + 18$ c. $x^2 - 3x - 18$ d. $x^2 + 3x - 18$

Factor each polynomial. Check using the FOIL method.

2. $x - 2x - 24$
3. $a^2 - 14a - 72$
4. $x^2 + 2xy + y^2$
5. $a - 15a + 24$
6. $2y^2 + 14y + 24$
7. $a^2 + 8a + 16$
8. $x^2 + 10x + 25$
9. $a^2 - 13a - 30$
10. $y^2 - 8y - 20$
11. $y^2 + y - 12$
12. $3x^2 - 12x + 9$
13. $a^2 + 10ab + 16b^2$
14. $x^2 - 6xy - 16y^2$
15. $4y^2 - 20y + 16$
16. $x^2 + 2x - 15$

**Teaching Notes:**

- Students need as much practice as possible to become comfortable factoring.
- Stress checking the answer! Especially the "oi" (outside + inside).

Answers: 1. a. $(x+6)(x+3)$ b. $(x-6)(x-3)$ c. $(x-6)(x+3)$ d. $(x+6)(x-3)$ 2. $(x-6)(x+4)$ 3. $(a-18)(a+4)$ 4. $(x+y)(x+y)$ 5. $(a-8)(a-3)$ 6. $2(y+3)(y+4)$ 7. $(a+4)(a+4)$ 8. $(x+5)(x+5)$ 9. $(a-15)(a+2)$ 10. $(y-10)(y+2)$ 11. $(y+4)(y-3)$ 12. $3(x-3)(x-1)$ 13. $(a+2b)(a+8b)$ 14. $(x-8y)(x+2y)$ 15. $4(y-4)(y-1)$ 16. $(x+5)(x-3)$

## Mini Lecture 6.3

Factoring Trinomials Whose Leading Coefficient is Not 1

**Learning Objectives:**

1. Factor trinomials by trial and error.
2. Factor trinomials by grouping.

**Examples:**

Factor each trinomial. Try both the trial and error method and grouping. If the trinomial is prime, so state. Show that the factorization is correct by multiplying the factors using the FOIL method.

| | | | |
|---|---|---|---|
| 1. | $6x^2+13x+6$ | 2. | $3x^2-4x-15$ |
| 3. | $5x^2+x-18$ | 4. | $6x^2+2x+1$ |
| 5. | $7x^2+15+2$ | 6. | $3x^2-5x-2$ |
| 7. | $3x^2+4x+1$ | 8. | $4x^2+4x-15$ |
| 9. | $18x^2-21x-9$ | 10. | $6x^2+7x+2$ |
| 11. | $8x^2+10x-3$ | 12. | $15x^2-19x+6$ |
| 13. | $6x^2-41x-7$ | 14. | $9x^2+18x+8$ |
| 15. | $3x^2+2x-5$ | 16. | $4x^2+2xy-6y^2$ |
| 17. | $6x^2-7xy-3y^2$ | 18. | $4x^2+4xy-3y^2$ |
| 19. | $6x^2+15x+9$ | 20. | $2x^2+5x-2$ |

**Teaching Notes:**

- When factoring $ax^2+bx+c$, it is important to know the sign combinations. If the polynomial is in the form $ax^2+bx+c$, then the factored form is ( + ) ( + ). If the polynomial is in the form $ax^2+bx-c$, then the factored form is ( + ) ( – ) or ( – ) ( + ).
- When using the trial and error method in factoring $ax^2+bx+c$, ALWAYS look for the greatest common factor first. If there is a GCF, factor it out first, then continue by finding 2 factors that equal $ax^2$ ( __ ) ( __ ). Then find two factors whose product equals $c$ ( __ ) ( __ ). Check the sum of the outside and inside product because it must equal $bx$ ( __ __ ) ( __ __ ). If no combination exists, the polynomial is prime.
- When using the grouping method to factor $ax^2+bx+c$, if $a \neq 1$, first multiply the leading coefficient, *a*, and the constant, *c*. Then find two factors of *ac* whose sum is *b*. Rewrite the middle term, *bx*, as a sum or difference using the two factors found. Use the grouping method discussed in 7.1.

Answers: 1. $(3x+2)(2x+3)$ 2. $(3x+5)(x-3)$ 3. $(5x-9)(x+2)$ 4. prime 5. $(7x+1)(x+2)$ 6. $(3x+1)(x-2)$ 7. $(3x+1)(x+1)$ 8. $(2x+5)(2x-3)$ 9. $3(3x+1)(2x-3)$ 10. $(3x+2)(2x+1)$ 11. $(4x-1)(2x+3)$ 12. $(5x-3)(3x-2)$ 13. $(6x+1)(x-7)$ 14. $(3x+4)(3x+2)$ 15. $(3x+5)(x-1)$ 16. $2(2x+3y)(x-y)$ 17. $(3x+y)(2x-3y)$ 18. $(2x+3y)(2x-y)$ 19. $3(x+1)(2x+3)$ 20. prime

## Mini Lecture 6.4
Factoring Special Forms

**Learning Objectives:**
1. Factor the difference of two squares.
2. Factor perfect square trinomials.
3. Factor the sum or difference of two cubes.

**Examples:**
Factor each polynomial completely.

1. a. $a^2-16$ b. $x^2-169$ c. $a^2-b^2$ d. $4x^2-9y^2$

2. a. $a^2+4a+4$ b. $y^2-10y+25$ c. $x^2+16x+64$ d. $4a^2-20a+25$

3. a. $x^3-1$ b. $y^3+27$ c. $8a^3+27b^3$ d. $16x^3-2$

   e. $64-y^3$ f. $a^6+b^6$ g. $5x^3-5y^3$ h. $a^3+125$

Factor completely. (Some may be prime.)

4. a. $16x^2+18y^2$ b. $x^4-81$ c. $a^2+12a+36$ d. $25x^2-16y^2$

   e. $3x^2+18x+27$ f. $125x^3-8$ g. $x^5+8x^2$ h. $27x^2-12$

**Teaching Notes:**
- Students must spend time learning to identify these special products.
- The difference of squares and perfect square trinomials seems to come fairly easily, but the cubes take more practice and simply must be memorized.
- A PST (**P**erfect **S**quare **T**rinomial) always results in BS (**B**inomial **S**quare or **B**inomials **S**ame).

Answers: 1. a. $(a+4)(a-4)$ b. $(x+13)(x-13)$ c. $(a+b)(a-b)$ d. $(2x+3y)(2x-3y)$
2. a. $(a+2)^2$ b. $(y-5)^2$ c. $(x+8)^2$ d. $(2a-5)^2$ 3. a. $(x-1)(x^2+x+1)$
b. $(y+3)(y^2-3y+9)$ c. $(2a+3b)(4a^2-6ab+9b^2)$ d. $2(2x-1)(4x^2+2x+1)$
e. $(4-y)(16+4y+y^2)$ f. $(a^2+b^2)(a^4-a^2b^2+b^4)$ g. $5(x-y)(x^2+xy+y^2)$
h. $(a+5)(a^2-5a+25)$ 4. a. $2(8x^2+9y^2)$ b. $(x^2+9)(x+3)(x-3)$ c. $(a+6)(a+6)$
d. $(5x+4y)(5x-4y)$ e. $3(x+3)(x+3)$ f. $(5x-2)(25x^2+10x+4)$ g. $x^2(x+2)(x^2-2x+4)$
h. $3(3x+2)(3x-2)$

**Mini Lecture 6.5**
A General Factoring Strategy

**Learning Objectives:**

1. Recognize the appropriate method for factoring a polynomial.
2. Use a general strategy for factoring polynomials.

**Examples:**

Factor the polynomials. Check your factorization by multiplying.

1. $9x^2 + 81$
2. $4x^2 - 64$
3. $27x^3 + 64y^3$
4. $x^3 - 1$
5. $81x^4 - 1$
6. $a^2 - 18a + 72$
7. $36a^2 + 66a + 24$
8. $12x^3 + 18x^2 - 30x^2 - 45x$
9. $2x^3 + 10x + x^2 + 5$
10. $2a^2 - 8ab + 12b^2$

**Teaching Notes:**

- Practice, practice, practice!
- When factoring, always look for a GCF first.
- Count the number of terms in the polynomial.
- If two terms – is it difference of squares, sum of two cubes, difference of two cubes?
- If three terms – is it a perfect square trinomial?
- With a trinomial, use trial and error or grouping method.
- If four or more terms, try factoring by grouping.
- Is the polynomial prime?

Answers: 1. $9(x^2 + 9)$ 2. $4(x+4)(x-4)$ 3. $(3x+4y)(9x^2 - 12xy + 16y^2)$ 4. $(x-1)(x^2 + x + 1)$
5. $(9x^2 + 1)(3x+1)(3x-1)$ 6. $(a-12)(a-6)$ 7. $6(3a+4)(2a+1)$ 8. $3x(2x-5)(2x+3)$
9. $(x^2 + 5)(2x+1)$ 10. $2(a^2 - 4ab + 6b^2)$

**Mini Lecture 6.6**

Solving Quadratic Equations by Factoring

**Learning Objectives:**

1. Use the zero product principle.
2. Solve quadratic equations by factoring.
3. Solve problems using quadratic equations.

**Examples:**

1. Solve each equation.

   a. $(x-3)(x+2)=0$ b. $(2x-5)(x-10)=0$ c. $(x+4)(x+7)=0$

2. Put each equation in standard form. Make sure the leading coefficient is positive.

   a. $4x^2+3x=10$ b. $5x=20-x^2$ c. $3x^2-6x=2x^2-11$

3. Solve each quadratic equation by factoring.

   a. $2x^2-3x-20=0$ b. $a^2-11a+30=0$ c. $2x^2-5x=12$

   d. $5x^2=2x$ e. $8x^3-2x^2=10x$ f. $x(14-x)=48$

   g. $4x^2-49=0$ h. $3x^2=15x$ i. $3a^2+7a-20=0$

**Teaching Notes:**

- Make sure students know what the standard form of a quadratic equation is and what the letters "*a,*" "*b,*" and "*c*" represent.
- Students need to be able to recognize a quadratic equation by the squared term.
- Students often want to shortcut the steps. Warn them not to take shortcuts when they should set each factor equal to zero and solve.

Answers: 1. a. $x=3$ $x=-2$ b. $x=\frac{5}{2}$ $x=10$ c. $x=-4$ $x=-7$ 2.a. $4x^2+3x-10=0$
b. $x^2+5x-20=0$ c. $x^2-6x+11=0$ 3. a. $x=-\frac{5}{2}$ $x=4$ b. a = 6 a = 5 c. $x=-\frac{3}{2}$ $x=4$
d. $x=0$ $x=\frac{2}{5}$ e. $x=0$ $x=\frac{5}{4}$ $x=-1$ f. $x=6$ $x=8$ g. $x=-\frac{7}{2}$ $x=\frac{7}{2}$ h. $x=0$ $x=5$
i. $a=\frac{5}{3}$ $a=-4$

# Mini Lecture 7.1

Rational Expressions and Their Simplification

**Learning Objectives:**

1. Find numbers for which a rational expression is undefined.
2. Simplify rational expressions.
3. Solve applied problems involving rational expressions.

**Examples:**

Find all the numbers for which the rational expression is undefined. If the rational expression is defined for all real numbers, so state.

1. a. $\dfrac{3x-21}{4x+20}$ b. $\dfrac{7x-49}{x^2+5x+6}$ c. $\dfrac{x+3}{3}$

Simplify.

2. a. $\dfrac{4x+24}{28x}$ b. $\dfrac{8x+56}{4x}$ c. $\dfrac{x^2+6x+5}{x+5}$

   d. $\dfrac{x^2+2x-15}{x^2+7x+10}$ e. $\dfrac{x+2}{4-x^2}$ f. $\dfrac{-c^2-3c}{c^2+2c-3}$

3. The rational expression $\dfrac{240}{r+20}$ describes the time, in hours, to travel 240 miles at a rate of ($r$ +20) miles per hour.
   a. Determine the value of $r$ that would cause the expression to be undefined.
   b. Find the time in hours if $r = 40$.

**Teaching Notes:**

- Rational expressions are quotients of two polynomials. They indicate division and division by zero is undefined. We must always exclude any value(s) of the variable that make a denominator zero.
- When simplifying rational expressions, first, factor the numerator and denominator completely, then divide both the numerator and the denominator by any common factors. A rational expression is simplified if its numerator and denominator have no common factors other than 1 and –1.
- When reducing rational expressions, only factors, not common terms, that are common to the entire numerator and the entire denominator can be divided out.

Answers: 1. a. $x=-5$ b. $x=-3$ and $x=-2$ c. defined for all real numbers 2. a. $\dfrac{x+6}{7x}$ b. $\dfrac{2x+14}{x}$ c. $x+1$ d. $\dfrac{x-3}{x+2}$ e. $\dfrac{1}{-x+2}$ or $\dfrac{1}{2-x}$ f. $\dfrac{-c}{c-1}$ 3. a. $r=-20$ b. time = 4 hours

## Mini Lecture 7.2
Multiplying and Dividing Rational Expressions

**Learning Objectives:**

1. Multiply rational expressions.
2. Divide rational expressions.

**Examples:**

Multiply as indicated.

1. a. $\dfrac{5}{x-3}\cdot\dfrac{x+2}{4}$ b. $\dfrac{9}{2x-8}\cdot\dfrac{x-4}{3}$ c. $\dfrac{4}{a-6}\cdot\dfrac{6}{a+4}$

2. a. $\dfrac{3x^3}{3x-6}\cdot\dfrac{x-2}{x^2}$ b. $\dfrac{4a^2+4a}{a^2-25}\cdot\dfrac{a^2-5a}{4a}$ c. $\dfrac{6x-12}{6x+12}\cdot\dfrac{3x+3}{12x-24}$

3. a. $\dfrac{x^2+5x+6}{x^2-x-6}\cdot\dfrac{2x^2-5x-3}{x^2+6x+9}$ b. $\dfrac{x^2+8x+16}{x^2+4x}\cdot\dfrac{x^2-x-6}{x^2-16}$ c. $\dfrac{x^2+5x-14}{x^2-8x+7}\cdot\dfrac{x-1}{x^2-49}$

Divide as indicated.

4. a. $\dfrac{3x+9}{x^2}\div\dfrac{6x+18}{x^3}$ b. $a^2-4a-12\div\dfrac{a+2}{a-6}$

   c. $\dfrac{x^2-3x-10}{x^2-8x+15}\div\dfrac{3x^2+2x-8}{x^2+x-12}$ d. $\dfrac{x^2+5x+1}{8x-8}\div\dfrac{x^2+5x+1}{x-1}$

   e. $\left(a^2+5a-24\right)\div\dfrac{(a-3)}{(a+8)}$

**Teaching Notes:**

- "When in doubt, factor it out." Factor first – before multiplying.
- Remind students that in order to divide fractions, (in this section, rational expressions) you must multiply by the reciprocal of the divisor.

Answers: 1. a. $\dfrac{5x+10}{4x-12}$ b. $\dfrac{3}{2}$ c. $\dfrac{24}{a^2-2a-24}$ 2. a. $x$ b. $\dfrac{a(a+1)}{a+5}$ c. $\dfrac{(x+1)}{4(x+2)}$
3. a. $\dfrac{2x+1}{x+3}$ b. $\dfrac{(x-3)(x+2)}{x(x-4)}$ c. $\dfrac{(x-2)}{(x-7)(x-7)}$ 4. a. $\dfrac{x}{2}$ b. $(a-6)^2$ c. $\dfrac{(x+4)}{(3x-4)}$
d. $\dfrac{1}{8}$ e. $(a+8)^2$

## Mini Lecture 7.3

Adding and Subtracting Rational Expressions with the Same Denominator

**Learning Objectives:**

1. Add rational expressions with the same denominator.
2. Subtract rational expressions with the same denominator.
3. Add and subtract rational expressions with opposite denominators.

**Examples:**

Add. Simplify if possible.

1. a. $\frac{3x-4}{7}+\frac{4x+11}{7}$ b. $\frac{x^2}{x^2-9}+\frac{9-6x}{x^2-9}$

Subtract, simplify if possible.

2. a. $\frac{6x+7}{x+2}-\frac{3x}{x+2}$ b. $\frac{4x^2+3x}{x+1}-\frac{-2x-1}{x+1}$

c. $\frac{2x^2+x-1}{x^2-2x-3}-\frac{x^2-x-2}{x^2-2x-3}$ d. $\frac{x^2}{x-2}-\frac{4}{2-x}$

Add, making sure to find a common denominator first and simplify if possible.

3. a. $\frac{x}{x-1}+\frac{1}{1-x}$ b. $\frac{x^2}{x-5}+\frac{25}{5-x}$

**Teaching Notes:**

- To add rational expressions with the same denominator, add numerators and place and sum over the common denominator. Simplify the answer if possible.
- To subtract rational expressions with the same denominator, subtract numerators and place the difference over the common denominator. Simplify the answer if possible.
- When subtracting numerators with a common denominator, make sure to subtract every term in that expression.
- When one denominator is the additive inverse of the other, first multiply either rational expressions by $\frac{-1}{-1}$ to obtain a common denominator.

Answers: 1. a. $x+1$ b. $\frac{x-3}{x+3}$ 2. a. $\frac{3x+7}{x+2}$ b. $4x+1$ c. $\frac{x+1}{x-3}$ d. $\frac{x^2+4}{x-2}$ 3. a. 1 b. $x+5$

**Mini Lecture 7.4**

Adding and Subtracting Rational Expressions with Different Denominators

**Learning Objectives:**

1. Find the least common denominator.
2. Add and subtract rational expressions with different denominators.

**Examples:**

Find the least common denominator for the rational numbers or rational expressions. Factor the denominators first, then build the least common denominator from those factors.

1. a. $\frac{1}{15}$ and $\frac{7}{24}$  b. $\frac{5}{4x^2}$ and $\frac{2x+3}{14x}$  c. $\frac{x+1}{x^2-16}$ and $\frac{2x-1}{x^2+6x+8}$

Rewrite each of the following as an equivalent expression with the given denominator.

2. a. $\frac{3}{8}=\frac{}{40x}$  b. $\frac{x^2}{7xy}=\frac{}{21x^3y^2}$  c. $\frac{5}{x+3}=\frac{}{(x+3)(x-2)}$

Add or subtract.

3. a. $\frac{2}{9}+\frac{5}{12}$  b. $\frac{3}{5x^2}+\frac{7}{10x}$  c. $\frac{3}{2a+4}+\frac{3}{a^2+2a}$

d. $\frac{x-3}{6}-\frac{x-1}{10}$  e. $\frac{y+3}{y-2}-\frac{4y-13}{y^2-5y+6}$  f. $\frac{2}{x^2-1}-\frac{5}{x^2+3x-4}$

g. $\frac{a-4}{a-3}+\frac{5}{a^2-a-6}$  h. $\frac{6}{y^2-9}-\frac{5}{y^2-y-6}$  i. $\frac{4x}{x^2+6x+5}-\frac{3x}{x^2+5x+4}$

**Teaching Notes:**

- Students may need to be reminded of factoring steps. It is very important to be able to factor quickly and completely.
- Students will find this concept easy if they can relate adding and subtracting rational expressions to adding and subtracting fractions.
- Watch the signs when subtracting!
- Students need to understand that the LCD is build with the factors of the denominators.

Answers: 1. a. 120  b. $28x^2$  c. $(x+4)(x-4)(x+2)$  2. a. $\frac{15x}{40x}$  b. $\frac{3x^4y}{21x^3y^2}$  c. $\frac{5x-1}{(x+3)(x-2)}$

3. a. $\frac{23}{36}$  b. $\frac{6+7x}{10x^2}$  c. $\frac{3}{2a}$  d. $\frac{x-6}{15}$  e. $\frac{y-2}{y-3}$  f. $\frac{-3}{(x+1)(x+4)}$  g. $\frac{a+1}{a+2}$

h. $\frac{1}{(y+3)(y+2)}$  i. $\frac{x}{(x+5)(x+4)}$

**Mini Lecture 7.5**
Complex Rational Expressions

**Learning Objectives:**

1. Simplify complex rational expressions by dividing.
2. Simplify complex rational expressions by multiplying by the LCD.

**Examples:**

Simplify by dividing; simplify, if possible.

1. a. $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{7}{12}+\frac{5}{6}}$ b. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}}$ c. $\dfrac{4+\frac{1}{x}}{4-\frac{1}{x}}$

Simplify by the LCD method; simplify, if possible.

1. a. $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{7}{12}+\frac{5}{6}}$ b. $\dfrac{4+\frac{1}{x}}{4-\frac{1}{x}}$ c. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}}$

**Teaching Notes:**

- Complex rational expressions are called complex fractions. They have numerators or denominators containing one or more rational expressions.
- One method for simplifying a complex rational expression is to add or subtract to get a single rational expression in the numerator and add or subtract to get a single rational expression in the denominator. Then divide by multiplying by the reciprocal of the term in the denominator.
- A second method for simplifying a complex rational expression is to multiply each term in the numerator and denominator by the least common denominator (LCD). This will produce an equivalent expression that does not contain fractions in the numerator or denominator.
- Both methods for simplifying complex rational expressions produce the same answer. See which method you prefer.

<u>Answers:</u> 1. a. $\dfrac{14}{17}$ b. $y+x$ c. $\dfrac{4x+1}{4x-1}$ 2. a.. $\dfrac{14}{17}$ b. $y+x$ c. $\dfrac{4x+1}{4x-1}$

## Mini Lecture 7.6
Solving Rational Equations

**Learning Objectives:**
1. Solve rational equations.
2. Solve problems involving formulas with rational expressions.

**Examples:**
What is the LCD in each problem?

1. a. $\frac{3}{2x}+\frac{1}{4}=\frac{5}{x}$ b. $\frac{5}{y}+3=\frac{2}{3y}$ c. $\frac{2x}{x-1}+\frac{3}{x}=5$

Solve. Clear each equation of fractions first.

2. a. $\frac{x}{2}-\frac{x}{3}=8$ b. $\frac{3}{x}-\frac{2}{3x}=\frac{14}{3}$ c. $\frac{y}{6}+\frac{y}{4}=5$

3. a. $\frac{a-2}{a-5}=\frac{a-3}{a+5}$ b. $\frac{x+3}{x+2}=\frac{x+2}{x+3}$ c. $\frac{5}{x+5}=\frac{3}{x+7}$

   d. $\frac{1-y}{1+y}=\frac{2}{3}$ e. $\frac{x-2}{x+2}=\frac{x-4}{x+4}$ f. $\frac{a+4}{a-2}=\frac{a+5}{a-3}$

4. a. $\frac{8}{x-3}-3=\frac{2-3x}{x+3}$ b. $\frac{4}{x-2}-\frac{2x-3}{x^2-4}=\frac{5}{x+2}$

   c. $\frac{3x^2-10}{2x^2-5x}-1=\frac{x}{2x-5}$ d. $\frac{3x-5}{x^2+4x+3}+\frac{2x+2}{x+3}=\frac{x-3}{x+1}$

   e. $4a-3=\frac{a+13}{a+1}$ f. $\frac{x-10}{5}-\frac{x-10}{3x}=0$

**Teaching Notes:**
- Students may need extra practice finding the LCD and may need to be reminded to Factor First.
- When all denominators are in factored form – list the restricted values. Restricted values are any numbers that would make any denominator zero.
- Some rational equations can be solved using cross products, but students need to be aware that the method can only be used when there is only one rational expression on each side of the equation.
- Students need to constantly be reminded to multiply each term or expression on both sides of the equation by the LCD to get ride of the fractions.
- Students must check solutions for restricted values.

Answers: 1. a. $4x$ b. $3y$ c. $x(x+1)$ 2. a. 48 b. $\frac{1}{2}$ c. 12 3. a. $\frac{25}{11}$ b. $-\frac{5}{2}$ c. –10 d. $\frac{1}{5}$ e. 0 f. –1 4. a. 19 b. 7 c. 2 d. –6 e. 2, –2 f. $\frac{5}{3}$, 10

## Mini Lecture 7.7
Applications Using Rational Equations and Proportions

**Learning Objectives:**

1. Solve problems involving motion.
2. Solve problems involving work.
3. Solve problems involving proportions.
4. Solve problems involving similar triangles.

**Examples:**

1. A boat travels 5 km upstream in the same amount of time that the boat covers 15 km downstream. The current in the stream moves at a speed of 2 km/h. What is the speed of the boat in still water?
2. John working alone can paint a room in 4 hours. His helper, Luke, would need 6 hours to do the job by himself. If they work together, how long will the paint job take to complete?
3. Six oranges cost \$1.86. At the same price, how much will 8 oranges cost?
4. A man can drive 490 miles in 7 hours. At the same rate, how far can he go in 12 hours?
5. A tree casts a shadow of 7.5 ft. At the same time, Elizabeth measures the length of her shadow which is 3 ft. If Elizabeth is 5.5 feet tall, how tall is the tree?

**Teaching Notes:**

- Time in motion equation: $t = \frac{d}{r}$ time traveled = $\frac{\text{distance traveled}}{\text{rate of travel}}$
- Work problem equation:
  fractional part of job done by one person + fractional part of job done by the second person = 1 job completed
- Similar Figures:
  Corresponding angles have the same measure and the ratios of the lengths of the corresponding sides are equal. In corresponding figures, the lengths of the corresponding sides are proportional. When triangles *ABC* and *DEF* are all similar then:
  $M\angle A = M\angle D,\ M\angle B = M\angle E,\ M\angle C = M\angle F$ $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$
- Direct Variation Equation: $y = kx$
  Where $k$ is the constant of variation, we say that $y$ varies directly as $x$.
- Inverse Variation Equation: $y = \frac{k}{x}$
  Where $k$ is the constant of variation, we say that y varies indirectly as $x$.
- With variation questions, write the equation from the English statements. Solve for $k$ and then substitute the value of $k$ back into the original equation to answer the original question.

Answers: 1. $\frac{5}{x-2} = \frac{15}{x+2}$, 4 km/h 2. $\frac{x}{4} + \frac{x}{6} = 1$, $2\frac{2}{5}$ hours 3. \$2.48 4. 840 miles

5. $\frac{x}{5.5} = \frac{7.5}{3}$, 13.75 feet

## Mini Lecture 7.8
Modeling Using Variation

**Learning Objectives:**
1. Solve direct variation problems.
2. Solve inverse variation problems.

**Examples:**

Which equations show direct variation?

1. a. $y = 4x$ b. $y = \frac{5}{8}$ c. $xy = 10$ d. $y = -7x$

Which equations show inverse variation?

2. a. $y = \frac{4}{\sqrt{x}}$ b. $y = \frac{10}{x}$ c. $y = \frac{x+2}{5}$ d. $y = 4$

Write a direction variation equation for each of the following and solve.

3. a. If $y$ varies directly to $x$ and $y$ is 10 when $x$ is 2, find $y$ when $x$ is 7.
   b. If $p$ varies directly to $m$, and $p$ is –32 when $m = 4$, find $p$ when $m$ is –3.
   c. If $d$ varies directly with $t$, and $d$ is 120 when $t$ is 2, find $d$ if $t$ is 3.5.

Write an inverse variation equation for each of the following and solve.

4. a. If $y$ varies inversely with $x$, and $y$ is 8 when $x$ is 5, find $y$ when $x$ is 2.
   b. If $b$ varies inversely with $a$, and $b$ is –2 when $a$ is –3, find $b$ when $a$ is 12.

**Teaching Notes:**
- Quantities can vary directly, inversely, or jointly.
- Direct Variation $y = kx$

  Inverse Variation $y = \frac{k}{x}$
- "$k$" is the constant of variation.

Answers: 1. $a$, $d$ 2. $a$, $b$ 3. a. $y = 35$ b. $p = 24$ c. $d = 210$ 4. a. $y = 20$ b. $b = \frac{1}{2}$

## **Mini Lecture 8.1**
Finding Roots

**Learning Objectives:**

1. Find square roots.
2. Evaluate models containing square roots.
3. Use a calculator to find decimal approximations for irrational square roots.
4. Find higher roots.

**Examples:**

Evaluate.

1. a. $\sqrt{49}$ b. $-\sqrt{100}$ c. $\sqrt{\frac{1}{36}}$ d. $\sqrt{9+16}$ e. $\sqrt{9}+\sqrt{16}$

Given the equation, $y=\dfrac{5}{4\sqrt{a}}$, solve for $y$ given that:

2. a. $a=9$ b. $a=25$ c. $a=49$

Use a calculator to approximate each expression and round to three decimal places. If the expression is not a real number, so state.

3. a. $\sqrt{30}$ b. $\sqrt{7}$ c. $\sqrt{11-1}$ d. $\sqrt{-8}$

Find the indicated root, or state that the expression is not a real number.

4. a. $\sqrt[3]{27}$ b. $\sqrt[5]{-1}$ c. $\sqrt[4]{-81}$ d. $\sqrt[5]{-32}$

   e. $\sqrt[6]{64}$ f. $-\sqrt[3]{64}$ g. $-\sqrt[4]{1}$ h. $\sqrt[5]{-32}$

**Teaching Notes:**

- The symbol $\sqrt{\ }$ is called the radical sign.
- The number under the radical sign is called the radicand.
- Together we refer to the radical sign and its radicand as a radical.
- The symbol $-\sqrt{\ }$ is used to denote the negative square root of a number.
- The square root of a negative number is not a real number. This also applies to any even root $\left(\sqrt[4]{-x},\ \sqrt[6]{-x},\ \sqrt[8]{-x}\ ...\right)$.
- Not all radicals are square roots.

Answers: 1. a. 7 b. –10 c. $\frac{1}{6}$ d. 5 e. 7 2. a. $\frac{5}{12}$ b. $\frac{1}{4}$ c. $\frac{5}{28}$ 3. a. 5.477 b. 2.646 c. 3.162 d. not a real number 4. a. 3 b. –1 c. not a real number d. –2 e. 2 f. –4 g. –1 h. –2

## Mini Lecture 8.2

Multiplying and Dividing Radicals

**Learning Objectives:**

1. Multiply square roots.
2. Simplify square roots.
3. Use the quotient rule for square roots.
4. Use the product and quotient rules for other roots.

**Examples:**

1. Multiply using the product rule.

 a. $\sqrt{3}\cdot\sqrt{5}$  b. $\sqrt{10}\cdot\sqrt{7}$  c. $\sqrt{6}\cdot\sqrt{\frac{1}{2}}$  d. $\sqrt{5}\cdot\sqrt{7}$

2. Simplify using the product rule.

 a. $\sqrt{20}$  b. $\sqrt{90}$  c. $\sqrt{48}$  d. $\sqrt{120}$

3. Simplify. (Look for a pattern) Assume all variables represent positive number only.

 a. $\sqrt{x^2}$  b. $\sqrt{x^3}$  c. $\sqrt{x^4}$  d. $\sqrt{x^5}$

 e. $\sqrt{x^6}$  f. $\sqrt{x^7}$  g. $\sqrt{x^8}$  h. $\sqrt{x^9}$

 i. $\sqrt{x^{10}}$  j. $\sqrt{x^{11}}$  k. $\sqrt{x^{12}}$

4. Simplify.

 a. $\sqrt{32x^2}$  b. $\sqrt{80y^6}$  c. $\sqrt{75x^7}$  d. $\sqrt{45x^4y^5}$

5. Multiply. Then simplify if possible.

 a. $\sqrt{6}\cdot\sqrt{2}$  b. $\sqrt{5x}\cdot\sqrt{10x}$  c. $\sqrt{7x^2}\cdot\sqrt{8x^3}$  d. $\sqrt{15y^4}\cdot\sqrt{5y^4}$

6. Simplify.

 a. $\sqrt{\frac{9}{25}}$  b. $\sqrt{\frac{8}{81}}$  c. $\sqrt{\frac{90x^2}{169}}$  d. $\sqrt{\frac{1}{49}}$

7. Simplify.

 a. $\sqrt[3]{24}$  b. $\sqrt[5]{1}$  c. $\sqrt[4]{48x^4}$  d. $\sqrt[3]{3}\cdot\sqrt[3]{6}$

**Teaching Notes:**

- Have students memorize perfect square numbers through 225 and perfect cubes through 216.
- Get as much out of the radicand as possible.
- Since radicals are unfamiliar to most students, it is important they see the relationship of squaring numbers and square roots, cubing numbers and cube roots, etc.

Answers: 1. a. $\sqrt{15}$ b. $\sqrt{70}$ c. $\sqrt{3}$ d. $\sqrt{35}$ 2. a. $2\sqrt{5}$ b. $3\sqrt{10}$ c. $4\sqrt{3}$ d. $2\sqrt{30}$
3. a. $x$ b. $x\sqrt{x}$ c. $x^2$ d. $x^2\sqrt{x}$ e. $x^3$ f. $x^3\sqrt{x}$ g. $x^4$ h. $x^4\sqrt{x}$ i. $x^5$ j. $x^5\sqrt{x}$ k. $x^6$
4. a. $4x\sqrt{2}$ b. $4y^3\sqrt{5}$ c. $5x^3\sqrt{3x}$ d. $3x^2y^2\sqrt{5y}$ 5. a. $2\sqrt{3}$ b. $5x\sqrt{2}$ c. $2x^2\sqrt{14x}$
d. $5y^4\sqrt{3}$ 6. a. $\frac{3}{5}$ b. $\frac{2\sqrt{2}}{9}$ c. $\frac{3x\sqrt{10}}{13}$ d. $\frac{1}{7}$ 7. a. $2\sqrt[3]{3}$ b. 1 c. $2x\sqrt[4]{3}$ d. $\sqrt[3]{18}$

**Mini Lecture 8.3**
Operations with Radicals

**Learning Objectives:**

1. Add and subtract radicals.
2. Multiply radical expressions with more than one term.
3. Multiply conjugates.

**Examples:**

Add or subtract as indicated.

1. a. $9\sqrt{2}+2\sqrt{2}$ b. $\sqrt{3x}-5\sqrt{3x}$
   c. $3\sqrt{8}+5\sqrt{18}$ d. $3\sqrt{27x}-8\sqrt{12x}$
   e. $2\sqrt{5}+4\sqrt{3}$

Multiply.

2. a. $\sqrt{3}\left(\sqrt{7}+\sqrt{5}\right)$ b. $\left(2+\sqrt{5}\right)\left(4+\sqrt{5}\right)$
   c. $\left(9+\sqrt{3}\right)\left(4-2\sqrt{3}\right)$ d. $\left(4+\sqrt{2}\right)\left(4-\sqrt{2}\right)$
   e. $\left(\sqrt{3}-\sqrt{5}\right)\left(\sqrt{3}+\sqrt{5}\right)$ f. $\left(\sqrt{x}+\sqrt{2}\right)^2$

**Teaching Notes:**

- Two or more square roots can be combined using the distributive property provided they have the same radicand.
- In some cases, radicals can be combined after they have been simplified.
- When multiplying radical expressions, distribute. This is similar to multiplying a monomial by a polynomial.
- When multiplying radical expressions use the FOIL method like multiplying binomials.
- When multiplying conjugates (expressions that involve the sum and difference of the same two terms), the FOIL method may be used or the special product formula. When using the FOIL method with conjugates the OI (outside & inside) will equal 0.

Answers: 1. a. $11\sqrt{2}$ b. $-4\sqrt{3x}$ c. $21\sqrt{2}$ d. $-7\sqrt{3x}$ e. cannot be combined
2. a. $\sqrt{21}+\sqrt{15}$ b. $13+6\sqrt{5}$ c. $30-14\sqrt{3}$ d. 14 e. –2 f. $x+2\sqrt{2x}+2$

**Mini Lecture 8.4**
Rationalizing the Denominator

**Learning Objectives:**

1. Rationalize denominators containing one term.
2. Rationalize denominators containing two terms.

**Examples:**

Multiply and simplify.

1. a. $\sqrt{2}\cdot\sqrt{2}$ b. $\sqrt{3}\cdot\sqrt{3}$ c. $\sqrt{4}\cdot\sqrt{4}$ d. $\sqrt{7}\cdot\sqrt{7}$
   e. $\sqrt[3]{9}\cdot\sqrt[3]{3}$ f. $\sqrt[3]{4}\cdot\sqrt[3]{2}$ g. $\sqrt[4]{4}\cdot\sqrt[4]{4}$

Rationalize each denominator.

2. a. $\sqrt{\dfrac{1}{2}}$ b. $\dfrac{2}{\sqrt{3}}$ c. $\sqrt{\dfrac{2}{3}}$ d. $\dfrac{\sqrt{2}}{\sqrt{8}}$
   e. $\dfrac{5}{\sqrt{5}}$ f. $\dfrac{2x}{\sqrt{10}}$ g. $\dfrac{2}{\sqrt{2x}}$ h. $\sqrt{\dfrac{5x}{7x}}$

State the conjugate of each of the following.

3. a. $4+\sqrt{2}$ b. $\sqrt{5}-\sqrt{3}$ c. $\sqrt{6}-3$ d. $\sqrt{7}+\sqrt{3}$

Multiply.

4. a. $\left(6+\sqrt{3}\right)\left(6-\sqrt{3}\right)$ b. $\left(\sqrt{11}+\sqrt{5}\right)\left(\sqrt{11}-\sqrt{5}\right)$ c. $\left(\sqrt{6}+2\right)\left(\sqrt{6}-2\right)$

Rationalize each denominator and write in simplest form.

5. a. $\dfrac{3}{\sqrt{3}-4}$ b. $\dfrac{5}{\sqrt{7}+\sqrt{3}}$ c. $\dfrac{\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ d. $\dfrac{\sqrt{x}-2}{\sqrt{x}+2}$ e. $\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

**Teaching Notes:**

- Remind students of the definition of a rational number. This will help them understand the meaning of "rationalizing the denominator."
- It may be helpful to discuss the special product of $(a+b)(a-b)$ with several examples to let students "see" again what happens to the middle term when the binomials are foiled.

<u>Answers:</u> 1. a. 2 b. 3 c. 4 d. 7 3. 3 f. 2 g. 2 2. a. $\dfrac{\sqrt{2}}{2}$ b. $\dfrac{2\sqrt{3}}{3}$ c. $\dfrac{\sqrt{6}}{3}$ d. $\dfrac{1}{2}$ e. $\sqrt{5}$ f. $\dfrac{x\sqrt{10}}{5}$ g. $\dfrac{\sqrt{2x}}{x}$ h. $\dfrac{\sqrt{35}}{7}$ 3. a. $4-\sqrt{2}$ b. $\sqrt{5}+\sqrt{3}$ c. $\sqrt{6}+3$ d. $\sqrt{7}-\sqrt{3}$ 4. a. 33 b. 6 c. 2 5 a. $\dfrac{-3\sqrt{3}-12}{13}$ b. $\dfrac{5\sqrt{7}-5\sqrt{3}}{4}$ c. $\dfrac{\sqrt{10}-2}{3}$ d. $\dfrac{x-4\sqrt{x}+4}{x-4}$ e. $\dfrac{5-\sqrt{21}}{2}$

**Mini Lecture 8.5**
Radical Equations

**Learning Objectives:**

1. Solve radical equations.
2. Solve problems involving square-root models.

**Examples:**

Solve each radical equation. If the equation has no solution, so state.

1. a. $\sqrt{4x+1}=3$ b. $\sqrt{3x+1}=4$

c. $\sqrt{x+9}-2\sqrt{x}=0$ d. $\sqrt{x}+3=0$

e. $\sqrt{4x+16}=x+5$ f. $\sqrt{x+2}=-6$

g. $\sqrt{2x+5}-1=4$ h. $2\sqrt{x}+4=1$

i. $\sqrt{x}-6=3$ j. $x=\sqrt{x+2}-2$

**Teaching Notes:**

- A radical equation is an equation in which the variable occurs in a square root, cube root, or any higher root.
- To solve a radical equation containing square roots, first arrange terms so that one radical is isolated on one side of the equation. Next, square both sides of the equation to eliminate the square root. Solve and ALWAYS check the answer in the original equation.
- There may be an extra solution(s) that does not check in the original equation. This solution(s) is/are called extraneous solutions.

Answers: 1. a. $x=2$ b. $x=5$ c. $x=3$ d. no solution e. –3 f. no solution g. 10 h. no solution i. 81 j. –2, –1

**Mini Lecture 8.6**
Rational Exponents

**Learning Objectives:**

1. Evaluate expressions with rational exponents.
2. Solve problems using models with rational exponents.

**Examples:**
Write each of the following in radical form first, then simplify.

1. a. $64^{\frac{1}{2}}$ b. $27^{\frac{1}{3}}$ c. $121^{\frac{1}{2}}$ d. $(-216)^{\frac{1}{3}}$ e. $81^{\frac{1}{4}}$

2. a. $64^{\frac{4}{3}}$ b. $25\frac{3}{2}$ c. $125^{\frac{2}{3}}$ d. $32^{\frac{2}{5}}$ e. $-27^{\frac{4}{3}}$

Simplify.

3. a. $6^{-2}$ b. $36^{-\frac{1}{2}}$ c. $36^{-\frac{1}{2}}$ d. $8^{-3}$ e. $8^{\frac{1}{3}}$ f. $8^{-\frac{1}{3}}$

4. a. $27^{-\frac{2}{3}}$ b. $100^{-\frac{3}{2}}$ c. $343^{-\frac{4}{3}}$ d. $256^{\frac{3}{4}}$ e. $64^{-\frac{1}{3}}$ f. $\left(\frac{1}{36}\right)^{-\frac{1}{2}}$

**Teaching Notes:**

- If a graphing calculator is being used in the class, it is **helpful** to show that $x^{\frac{1}{n}}$ is the same as the $\sqrt[n]{x}$ using number values.
- Stress to students that the denominator of a rational exponent is the index of the corresponding radical expression.
- When the numerator of a rational exponent is not 1, the numerator is the power to which the radical is raised. It is usually easier to simplify it this way, but it is possible to raise the radicand to the power instead.
- When the exponent is negative, write the base as its reciprocal, and raise to the positive power.

Answers: 1. a. $\sqrt{64}=8$ b. $\sqrt[3]{27}=3$ c. $\sqrt{121}=11$ d. $\sqrt[3]{-216}=-6$ e. $\sqrt[4]{81}=3$
2. a. $\left(\sqrt[3]{64}\right)^4=256$ b. $\left(\sqrt{25}\right)^3=125$ c. $\left(\sqrt[3]{125}\right)^2=25$ d. $\left(\sqrt[5]{32}\right)^2=4$ e. $-\left(\sqrt[3]{27}\right)^4=-81$ 3. a. $\frac{1}{36}$ b. 6 c. $\frac{1}{6}$ d. $\frac{1}{521}$ e. 2 f. $\frac{1}{2}$ 4. a. $\frac{1}{9}$ b. $\frac{1}{1000}$ c. $\frac{1}{2401}$ d. 64 e. $\frac{1}{4}$ f. 6

## Mini Lecture 9.1
Solving Quadratic Equations by the Square Root Property

**Learning Objectives:**
1. Solve quadratic equations using the square root property.
2. Solve problems using the Pythagorean Theorem.
3. Find the distance between two points.

**Examples:**

1. Solve each quadratic equation by the square root property. If possible, simplify radicals or rationalize denominators.

   a. $x^2 = 25$ b. $3x^2 = 27$ c. $3x^2 - 5 = 0$ d. $(x-2)^2 = 16$ e. $(x+4)^2 = 5$

2. Solve each quadratic equation by first factoring the perfect square trinomial on the left. Then apply the square root property. Simplify radicals, if possible.

   a. $x^2 + 6x + 9 = 64$ b. $x^2 + 8x + 16 = 11$

3. Use the Pythagorean Theorem to solve the following.

   If a ladder is angled against a home and it is 6 ft. from the base of the home and reaches 8 ft. high on the side of the home, how tall is the ladder?

4. Find the distance between the given 2 points. Express answers in simplest radical form and if necessary, round to two decimal places.

   a. $(-3, 6)$ and $(1,\ 5)$ b. $(5, -3)$ and $(9,\ -6)$

**Teaching Notes:**
- The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If the lengths of the legs are *a* and *b,* and the hypotenuse has length *c*, then $a^2 + b^2 = c^2$.
- The Distance Formula is used to find the distance between two points in the rectangular coordinate system $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ given 2 points $(x_1, y_1)(x_{2,} y_2)$. Remind students about the subscript numbers and their meaning.

Answers: 1. a. $\pm 5$ b. $\pm 3$ c. $\dfrac{\pm\sqrt{15}}{3}$ d. 6, –2 e. $-4 \pm \sqrt{5}$ 2. a. –11 ,5 b. $-4 \pm \sqrt{11}$ 3. 10 ft. 4. a. $\sqrt{17}$ b. 5

## Mini Lecture 9.2

Solving Quadratic Equations by Completing the Square

**Learning Objectives:**

1. Complete the square of a binomial.
2. Solve quadratic equations by completing the square.

**Examples:**

1. Complete the square for each binomial by adding "one-half the coefficient of the $x$ term squared." Then factor the resulting perfect square trinomial.

a. $x^2 + 2x +$ __________ = __________ b. $x^2 + 8x +$ __________ = __________

c. $x^2 - 16x +$ __________ = __________ d. $x^2 + 10x +$ __________ = __________

e. $x^2 - 14x +$ __________ = __________ f. $x^2 + 3x +$ __________ = __________

g. $x^2 - 6x +$ __________ = __________

2. Solve each quadratic equation by completing the square.

a. $x^2 = 11x - 30$ b. $x^2 + 2x - 8 = 0$ c. $x^2 = 4x - 1$

d. $16x^2 = 8x + 3$ e. $x^2 + 12x + 12 = 0$ f. $x^2 + \frac{8}{3}x = 1$

g. $x^2 - 6x = 3$

**Teaching Notes:**

- Remind students that in order to complete the square the leading coefficient must be 1 and there must be an $x^2$ term and an $x$ term.
- When using completing the square to solve a quadratic equation, the variable terms must be on one side of the equation and the constant term on the other side before starting the process.

Answers: 1. a. $x^2 + 2x + 1 = (x+1)^2$ b. $x^2 + 8x + 16 = (x+4)^2$ c. $x^2 - 16x + 64 = (x-8)^2$ d. $x^2 + 10x + 25 = (x+5)^2$ e. $x^2 - 14x + 49 = (x-7)^2$ f. $x^2 + 3x + \frac{9}{4} = (x + \frac{3}{2})^2$ g. $x^2 - 6x + 9 = (x-3)^2$ 2. a. 6, 5 b. 4, –2 c. $2 \pm \sqrt{3}$ d. $\frac{3}{4}, -\frac{1}{4}$ e. $-6 \pm 2\sqrt{6}$ f. $\frac{1}{3}, -3$ g. $3 \pm 2\sqrt{3}$

## **Mini Lecture 9.3**
The Quadratic Formula

**Learning Objectives:**

1. Solve quadratic equations using the quadratic formula.
2. Determine the most efficient method to use when solving quadratic equations.
3. Solve problems using quadratic equations.

**Examples:**

1. Solve each equation using the quadratic formula.

   a. $3x^2 + 10x + 3 = 0$

   b. $x^2 = 5x + 2$

   c. $x^2 + 3x = -1$

2. Solve each equation by the <u>method of your choice.</u> Simplify irrational solutions, if possible.

   a. $2x^2 = 40$
   b. $(x+4)^2 = 8$
   c. $3x^2 - 4x + 1 = 0$
   d. $2x^2 + 7x + 4 = 0$
   e. $4x^2 + 2x - 5 = 0$

3. A football is kicked straight up from a height of 2 feet with an initial speed of 50 ft. per second. The formula $h = -16t^2 + 50t + 2$ describes the ball's height above the ground, $h$, in feet, $t$ seconds after it is kicked. Using your calculator, find when the ball will hit the ground. Round to the nearest tenth of a second.

<u>Answers:</u> 1. a. $-\frac{1}{3}, -3$ b. $\frac{5 \pm \sqrt{33}}{2}$ c. $\frac{-3 \pm \sqrt{5}}{2}$ 2. a. $x = \pm 2\sqrt{5}$ b. $x = -4 \pm 2\sqrt{2}$
c. $x = \frac{1}{3}, x = 1$ d. $\frac{-7 \pm \sqrt{17}}{4}$ e. $\frac{-1 \pm \sqrt{21}}{4}$ 3. 3.2 seconds

## Mini Lecture 9.4

Imaginary Numbers as Solutions of Quadratic Equations

**Learning Objectives:**

1. Express square roots of negative numbers in terms of *i*.
2. Solve quadratic equations with imaginary solutions.

**Examples:**

Write as a multiple of *i*.

1. a. $\sqrt{-64}$ b. $\sqrt{-4}$ c. $\sqrt{-10}$ d. $\sqrt{-20}$ e. $\sqrt{-40}$ f. $\sqrt{-75}$

Solve each quadratic equation using the square root property.

2. a. $(x-2)^2=-16$ b. $(x+3)^2=-1$ c. $x^2+4=0$

Solve each quadratic equation using the quadratic formula.

3. a. $x^2+4x+7=0$ b. $2x^2+18=0$ c. $6x^2+8x=5$

   d. $3x^2+16x+5=0$ e. $3x^2+x=-3$

**Teaching Notes:**

- When simplifying the square root of –1, it is helpful to write the "*i*" to the left of the radical so it does not look like the "*i*" is under the radical.
- Always simplify the $\sqrt{-1}$ first.
- Remind students that in order to use the square root property to solve a quadratic equation, there must be a perfect square containing the variable isolated on 1 side of the equation.
- Students need to be cautioned again about watching the signs very carefully when using the quadratic formula.

Answers: 1. a. $8i$ b. $2i$ c. $i\sqrt{10}$ d. $2i\sqrt{5}$ e. $2i\sqrt{10}$ f. $5i\sqrt{3}$ 2. a. $2\pm 4i$ b. $-3+i$ c. $\pm 2i$
3. a. $-2\pm i\sqrt{3}$ b. $3i, -3i$ c. $\dfrac{-4\pm\sqrt{46}}{6}$ d. $-\dfrac{1}{3}, -5$ e. $\dfrac{-1\pm i\sqrt{35}}{6}$

**Mini Lecture 9.5**
Graphs of Quadratic Equations

**Learning Objectives:**

1. Understand the characteristics of graphs of quadratic equations.
2. Find a parabola's intercepts.
3. Find a parabola's vertex.
4. Graph quadratic equations.
5. Solve problems using a parabola's vertex.

**Examples:**

1. Consider the quadratic equation: $y = x^2 + 4x + 3$.

   a. Will the parabola open upward or downward?
   b. Complete the table of values and graph the parabola.

| $x$ | $y = x^2 + 4x + 3$ | $(x, y)$ |
|---|---|---|
| –4 | | |
| –3 | | |
| –2 | | |
| –1 | | |
| 0 | | |
| 1 | | |
| 2 | | |

   c. Find the *x*-intercepts.
   d. Find the *y*-intercepts.
   e. Find the vertex.

2. Complete the same steps a, b, c, d, e, from number 1 for the equation: $y = -x^2 - 2x - 1$.

**Teaching Notes:**

- The graph of quadratic equation $y = ax^2 + bx + c$, $a \neq 0$ is called a parabola. If $a$ is positive, the parabola opens upward. If $a$ is negative, the parabola opens downward.
- The vertex, or turning point, of the parabola is referred to as the maximum if the parabola opens downward and the minimum if the parabola opens upward.
- Parabolas are symmetric with respect to a line known as the axis of symmetry that runs through the vertex. The two halves match exactly.
- To find the *x*-intercepts, let *y* = 0. Then factor or use the quadratic formula to solve for *x*.
- To find the *y*-intercept, let *x* = 0 and solve for *y*.

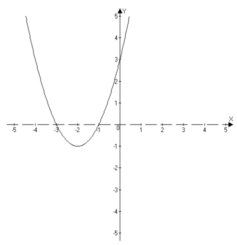
- To find the vertex of a parabola with the equation, $y = ax^2 + bx + c$, first solve for $x$. $x = -\frac{b}{2a}$. Then substitute the value of $x$ into the parabola's equation and evaluate.
- After plotting the vertex and intercepts, make sure to connect with a smooth curve.

Answers:

1. a. up

   b.

| | |
|---|---|
| –4 | (–4, 3) |
| –3 | (–3, 0) |
| –2 | (–2, –1) |
| –1 | (–1, 0) |
| 0 | (0, 3) |
| 1 | (1, 8) |
| 2 | (2, 15) |



   c. $x$-intercepts (–3, 0)(–1, 0)

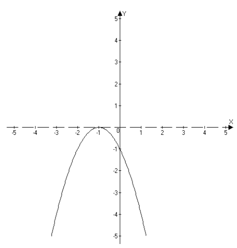
   d. $y$-intercept (0, 3)

   e. vertex (–2 , –1)

2. a. down

   b.

| | |
|---|---|
| –4 | (–4, –9) |
| –3 | (–3, –4) |
| –2 | (–2, –1) |
| –1 | (–1, 0) |
| 0 | (0,–1) |
| 1 | (1, –4) |
| 2 | (2, –9) |



   c. $x$-intercept (–1, 0)

   d. $y$-intercept (0 , –1)

   e. vertex (–1 , 0 )

## Mini Lecture 9.6

Introduction to Functions

**Learning Objectives:**

1. Find the domain and range of a relation.
2. Determine whether a relation is a function.
3. Evaluate a function.
4. Use the vertical line test to identify functions.
5. Find function values for functions that model data.

**Examples:**

Determine whether relation is a function. Give the domain and range for each relation.

1. a. $\{(3,4),(4,5),(5,6)\}$ b. $\{(2,7),(4,8),(2,10),(8,12)\}$

   c. $\{(-2,3),(-1,3),(2,3),(3,3)\}$ d. $\{(5,-2),(-3,6),(0,-2)\}$

   e. $\{(7,0),(6,5),(5,7),(7,9)\}$

2. Evaluate the functions at the given values.

   $f(x)=3x-5$

   a. $f(3)$ b. $f(-2)$ c. $f(0)$

3. $g(x)=x^2-x+2$

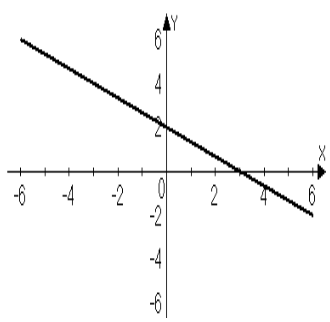
   a. $g(4)$ b. $g(0)$ c. $g(-2)$

4. $h(x)=-2x^2+x$

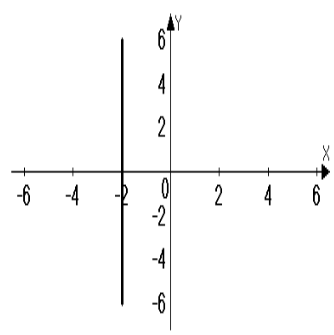
   a. $h(0)$ b. $h(-3)$ c. $h(1)$

5. Use the vertical line test to identify graph in which $y$ is a function of $x$.

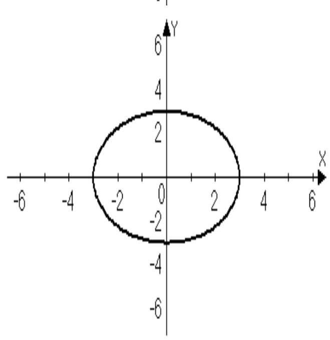
a.




b.




c.



**Teaching Notes:**

- Students should understand that every function is a relation BUT not every relation is a function.
- The domain of a relation or a function is all the *x*-values or possible *x*-values.
- The range of a relation or a function is all the *y*-values or possible *y*-values.
- A function is a relation in which **no two *x* values are the same**.
- "*f*(*x*)" and "*y*" are like math synonyms.
- *f*(*x*) indicates a function.

Answers: 1. a. function; domain {3, 4, 5}; range {4, 5, 6}
b. not a function; domain {2, 4, 8}; range {7, 8, 10, 12}
c. function; domain {–2, –1, 2, 3}; range {3}
d. function; domain }5, –3, 0}; range {–2, 6}
e. not a function; domain {7, 6, 5}; range {0, 5, 7, 9}
2. a. $f(x)-6x+7$ b. $f(x)=2x^2+5$ 3. a. 4 b. –11 c. –5 4. a. 14 b. 2 c. 8 5. a. 0
b. –21 c. –1 5. a. function b. not a function c. not a function